Q. F. S. F. Q.

CONTINUATIO DISSERTATIONIS

DE

RETROGRADATIONE

ALIISQUE APPARENTIIS MOTUS

PLANETARUM,

QUAM

Suffrag. Amplisſ. Senat. Philoſ. in Reg. Acad. Aboënſi,

Publico examini modeſte ſubmittunt,

AUCTOR

MAGNUS JACOBUS ALOPAEUS

ET

RESPONDENS

JACOBUS JOHANNES URSINUS,

WIBURGENSES.

IN AUDITORIO MAJORI DIE XIV. JUNII

ANNI MDCCLXIX.

HORIS A. M. CONSUETIS.

ABOÆ,

Impreſſit JOH. CHRISTOPH. FRENCKELL.

I. N. J.

Praxin l. adplicationem formularum, quas pro adparentiis quibusdam motus Planetarum in dissertatione, cujus hæc est continuatio, exhibuimus, tuis B. L. subjicere oculis nunc mihi licet. Quamvis vero nulla præcipua elegantia sese commendet hæc opella, quæ præter numeros ac molestam computandi rationem vix quidquam continet; prorsus injucundam tamen meam operam haud fore eo lubentius spero, quo magis mihi persuasum est, non prostare apud scriptores, quæ his pagellis præstiti, scil. in primis Stationes, Progressiones atque Retrogradationes Planetarum primariorum, e Tellure spectatorum, calculo, quam fieri potuit adcurato, erutas. Quin igitur B. L. juveniles hos conatus sit æqua atque benigna lance pensitaturus, nulli dubitamus.

§. XIV.

$$\frac{d^3z}{z\,dx} + P\frac{dz\,dx^2}{z} + Q\,dx^3 = 0$$

$$\frac{dz}{z} = dy,\ \frac{d^2z}{z} = d^2y + dy^2,\ \frac{d^3z}{dz} = d^3y + 3\,dy\,d^2y + dy^3$$

$$d^3y + 3\,dy\,d^2y + dy^3 +$$

$$\frac{d^2z}{z\,dx^2} + \varphi\frac{dz}{z\,dx} + \psi = 0$$

$$\frac{d^3z}{z\,dx^2} - \frac{dz\,d^2z}{z^2\,dx^2} + \frac{d\varphi\,dz}{z\,dx} + d\psi$$

$$-\frac{d^3z}{z\,dx^2} + \frac{\varphi\,d^2z}{z\,dx} + \frac{R\varphi\,dz}{z} + R\psi\,dx$$

$$+\frac{R\,d^2z}{z\,dx} \qquad -\frac{P\,dz}{z} \qquad -Q\,dx$$

$$\cancel{P\,dz}$$

$$\frac{d\varphi}{\varphi\,dx} - \frac{P}{\varphi} = \frac{d\psi}{\psi\,dx} - \frac{Q}{\psi};$$

$$\frac{dz}{z} = \varphi\,dx + R\,dx =$$

$$\frac{d^2z}{z} = \frac{dz^2}{z^2} + (d\varphi + dR)\,dx = -\varphi\frac{dz\,dx}{z} - \psi\,dx^2$$

$$= -\varphi^2dx^2 - \varphi R\,dx^2$$

$$\cancel{dx} \quad dx = N^{sdy}\,dv \qquad \frac{dy}{dx} = N^{\int v\,dx}$$

$$p = \frac{dy}{dx} = \frac{dy}{N^{sdy}\,dv} \qquad \frac{dy}{dx} = v\,dx\,N^{\int v\,dx}$$

$$dp = q\,dx = \frac{d^2y - s\,dy^2}{N^{sdy}\,dv} \qquad dx = \frac{dv}{z}N^{\int\frac{v\,dx}{z}}$$

$$q = \frac{d^2y - s\,dy^2}{N^{2sdy}\,dv^2} \qquad \frac{dy}{dx} = N^{\int\frac{v\,dv}{z}}$$

$$d^2y - s\,dy^2 = X\,dv^2 \qquad \frac{d^2y}{dy} = \frac{dv}{z}N^{\int\frac{dv}{z}}$$

$$d^2y = \frac{dv^2}{z^2}$$

$\frac{d^2z}{z} + X\,dx^2 = 0$, $\frac{dz}{z} = dy$, $\frac{d^2z}{z} = d^2y + dy^2$

$d^2y + dy^2 + X\,dx^2 = 0$; $dy = \frac{dv}{z}$ $\frac{dy}{dx} = N^{\int\frac{vdv}{z}}$

$\frac{d^2y}{dx} = \frac{v\,dv}{z}$ $\quad d^2y = v\,dy^2$

$\frac{dy^2}{dx} = \frac{dv}{z}N^{\int\frac{vdv}{z}}$: $\overline{v+1}\,\frac{dy^2}{dx^2} + X = 0$

$\overline{v+1}\cdot N^{2\int\frac{vdv}{z}} + X = 0$

$dx = \frac{dv}{zN^{\int\frac{vdv}{z}}}$; $\quad dx = N^{sx}\,dv$

$\frac{dy}{dx} = \frac{dy}{N^{sx}\,dx}$

$\frac{d^2y}{dx^2}$ $\frac{d.\frac{dy}{dx}}{dx} = \frac{d^2y - s\,dy\,dx}{N^{2sx}}$ $\quad \frac{dx\,d^2y}{dy} + dx^2 + Y\,dy^2 = 0$

$dx\,d^2y + dy\,dx^2 + Y\,dy^3 = 0$

$dx\,d.\frac{dy}{dx} = -\frac{dy\,d^2x}{dx}$

$N^{sy^n} = z$

$d^2y = 0$ $\quad n s y^{n-1}\,dy\,N^{sy^n} = dz$

$N^{sy^n}(n.\overline{n-1}\,s\,y^{n-2} + n^2s^2y^{2n})\,dy^2 = d^2z$

$\frac{d^2z}{z} = (n.\overline{n-1}\,s\,y^{n-2} + n^2s^2y^{2n})\,dy^2$

§. XIV.

Fecimus quidem mentionem (§. 5. seq.) Planetarum Heterodromorum & de his signa inferiora, quando in Formulis nostris ambigua occurrunt, valere voluimus. Cum vero, in nostro saltem Systemate, omnes Planetæ sint *Homodromi* seu versus unam eandemque plagam moveri observentur, in eo tantum nostra jam versabitur opera, ut formulas, quales solis homodromis competunt, proferamus & ad earum ductum omnem nostrum calculum instituamus.

Observandum quoque est, nos non iis uti formulis, quas pro stationibus generatim elicuimus (§. VII.). Prolixum hoc ipsum foret & præterea minus necessarium. Formulæ enim, quæ §. X. exstant & a legibus naturæ pendent, in praxi sunt commodissimæ. Nemo tamen forte mirabitur si durationes Stationum, Directionum & Retrogradationum &c. æque ac arcus interea a Planetis emensi & per formulas nostras computati, observationibus, quas nobis videre contigit & suo loco indicabimus, non sint adcurate conformes. Præterquam enim quod his observationibus ne tuto quidem confidere possim, fieri quoque haud potest, ut valores calculo inventi, illis, summa licet accuratione institutis, exacte convenirent. Nam Planetæ re vera motu inæquabili orbitas ad se invicem inclinatas & quam proxime Ellipticas (§. II.) describunt; quo fit, ut Retrogradationes aliæque motuum adparentiæ cujusque Planetæ aliis atque aliis temporibus deprehendantur inæquales (*a*)

Calculo igitur, secundum formulas in simplicissima hypothesi (§. II.) orbitarum Circularium in communi plano positarum inventas, inito, non potest non aliqua saltim esse differentia inter valores sic elicitos & eos, qui ex observationibus haberi possunt. Quod denique durationes *Stationum* a Viris Celeberrimis passim observatas & indicatas speciatim concernit, illas prorsus laxas atque indeterminatas eam ob caussam esse contendimus, quod non determinaverint, intra quem visibilem celeritatis gradum Planeta Stationarius vocetur.

(a) Cfr. De La Lande Astr. §. 835 p. 391.

§. XV.

Ne computandi regulas cognituro Lectori benevolo opus sit ad priorem Dissertationis partem recurrere: formulas, quibus utar, inde excerpere & h. l. exhibere lubet.

Designantibus S Solem, P Planetam a Sole remotiorem, ℘ Soli propiorem; T Tempus periodicum Planetæ P; Z, Y, U angulos ad S, P, ℘ respective △:li rectilinei SP℘; $n : 1 :: SP : S℘$; $m : 1 :: T$: Tempus periodicum Planetæ ℘; c velocitatem Heliocentricam Planetæ P; v velocitatem adparentem ceu geocentricam Planetæ alterius ex altero spectati, & quidem directi, at $-v$ Retrogradi; $b = \frac{v}{c}$: sunt

1) Pro *Stationibus* (p. 24) Cotang $Y = \pm\sqrt{n . \overline{n+1}}$, Cotang $U = -\frac{\sqrt{n+1}}{n}$, Sin $Z = \frac{n-1}{m+1} . \sqrt{n+1}$, & Cos $Z = \frac{n(m+n)}{m(m+1)}$. Atque adhibitis his ipsorum Z, Y, U valoribus:

2) *Duratio Retrogradationis* $= \frac{Z . T}{m-1 . 180^\circ}$, *Directionis*

$$= \frac{(180^\circ - Z)T}{(m-1)180^\circ} = \frac{(U+Y)T}{(m-1)\,180^\circ}$$ (§. VII. Cor. 3.).

3) Motu autem *Retrogrado* Planeta superior (§. VII. Cor. 4.)

Ar-

Si visibilis, tunc tempore = 1" motus apparens sub ang. optico fiat > 17" efr. Ducoi Opp. §127.

$t = 4'$ $\quad arc = 1^\circ$ circa

intra $\dot{} = 1^d$ $\quad \dot{=} 360^\circ = 1$ tot. orbita.

Simplicius $\cos z = \frac{m+n}{m+1.\sqrt{n}}$, ob $\frac{n}{m} = \sqrt{\frac{n^2}{m^2}} = \sqrt{\frac{n^2}{n^3}} = \frac{1}{\sqrt{n}}$.

$y = \frac{dz}{z\,dx} + \alpha x^m; \quad dy = \frac{d^2z}{z\,dx} - \frac{dz^2}{z^2 dx} + m\alpha x$

$\frac{d^2z}{z\,dx} + 2\alpha x^m \frac{dz}{z} + \alpha^2 x^{2m} dx + m\alpha x^{m-1} dx + a x^n dx = 0$

$+\frac{13}{7} : -\frac{8}{5} \qquad +\frac{7}{3} : -\frac{5}{3} \qquad +\frac{7}{7} - 4 \qquad \frac{13}{7}$

$+$

$z = \mathcal{N}^{\delta x^r} \quad \frac{dz}{z} = r\delta x^{r-1} dx, \quad \frac{d^2z}{z} = \overline{r-1}\, r\delta x^{r-2} dx + r^2\delta^2 x^{2r-2} dx$

$\frac{-2}{-1} = 2$

$0 = \overline{r-1}\, r\delta x^{r-2} + r^2\delta^2 x^{2r} + 2r\delta\alpha x^{m+r-1} + \alpha^2 x^{2m}$

$+ m\alpha x^{m-1} + a x^n \qquad \cancel{2}\, r\delta = \alpha \quad +\frac{5}{2} : -\frac{3}{2}$

$2r \qquad \cancel{+\frac{3}{2} : -\frac{1}{2}} \qquad \frac{5-12}{3}$

$m = r-1, \; 2m = m+r-1 \qquad \frac{+3}{-2} \quad +\frac{3}{2} - 4 = -\frac{5}{2} \qquad \frac{7}{3}$

$m-1 = r-2; \qquad \frac{1}{4}$

$-1; -3; -\frac{3}{2}; -\frac{5}{2}; -\frac{5}{3}; -\frac{7}{3}; -\frac{7}{5}$

$n = m; \; n = -\frac{m}{m+1}; \; n = -m-4$

$-3; -\frac{5}{3}; -\frac{7}{5};$

$-1; -\frac{3}{2}; -\frac{5}{2}; -\frac{7}{3}; -\frac{13}{4}$

$n = -1; -3; -\frac{5}{2}; -\frac{7}{3}; -\frac{9}{4}; -\frac{11}{5}; \&c \quad = -\frac{2i+1}{i}$

$-\frac{3}{2}; -\frac{5}{3}; -\frac{7}{4}; -\frac{9}{5}; \quad = -\frac{2i-1}{i}$

$+\frac{5}{2} : -\frac{3}{2} \qquad n = -\frac{2r \pm 1}{r}$

$+\frac{5}{3} - 4 = \frac{5-12}{3}$

$+\frac{7}{3} : -\frac{4}{3};$

$\frac{7-16}{4} \quad +\frac{9}{4} : -\frac{5}{4}$

$+7-20$

Arcum $= 2Y - \frac{2Z}{m-1}$, *Directo Arcum* $= 2Y - \frac{2Z}{m-1} + \frac{360°}{m-1} = 2Y + \frac{2(U \mp Y)}{m-1}$ percurrere videtur.

4) Periodus Phænomenorum (§. VI. β) $= \frac{T}{m-1}$. Adeoque *Tempus* inter *Oppositionem* Heliocentricam (p. 7) & proxime sequentem *Conjunctionem* intercedens $= \frac{T}{2(m-1)}$, quo quidem superior (§. VI. α) arcum $\frac{180°}{m-1}$, Inferior arcum $\frac{m.\,180°}{m-1}$ motu vero descripsit.

$Z = 180°$

$Z = 0,\ U = 180°$

5) In *Oppositione* est $v = \frac{m+n}{n+1}.c$, in *Conjunctione* $v = -\frac{m-n}{n-1}.c$; & quidem utraque *velocitas* in suo genere *maxima*.

6) Dato vel ad libitum assumto aliquo angulorum Z, Y, U (*), reperitur (commodissime quidem per formulam p. 26. lin. 1. 2.) *adparens Planetæ velocitas* $v = c\left(1 + \frac{m-1}{2n} \cdot \frac{\text{Sin}\,2U}{\text{Sin.}\,Z}\right)$ seu $v = c\left(1 + \frac{m-1}{n} \cdot \frac{\text{Sin.U.Cos U}}{\text{Sin.}\,Z}\right)$

7) Data autem Planetæ velocitate adparente seu $\frac{v}{c} = b$: erunt (p. 21) $\text{Cos}\,Y = \frac{m-b}{n} \cdot \sqrt{\frac{nn-1}{\overline{m-1}.\overline{m+1-2b}}}$,

Cos U

Cos U $= -(1-b)\sqrt{\dfrac{m-1}{m-1 \,.\, m+1-2b}}$ & Cos. Z $= \dfrac{m-b}{n(m+1-2b)} + \dfrac{n(1-b)}{m+1-2b}$. Hinc

8) Si brevitatis caussa ponantur $\dfrac{m-b}{n} = A$, $n.\overline{1-b} = B$, $\dfrac{m+b}{n} = \mathfrak{A}$, $n.\overline{1+b} = \mathfrak{B}$; & quærantur Cos Z $= \dfrac{A+B}{m+1-2b}$, Cos $\mathfrak{Z} = \dfrac{\mathfrak{A}+\mathfrak{B}}{m+1+2b}$: mora temporis, qua velocitas adparens Planetæ (partim directi partim retrogradi) non excedit datum celeritatis gradum b, erit (p. 22. Cor. 3) $= \dfrac{Z-\mathfrak{Z}}{m-1.\ 360^\circ}$. T. Denique

9) Sumto ζ = differentiæ angulorum, quorum Cosinus sunt $\dfrac{1}{n}$ & $\dfrac{n(m+n)}{m(m+1)}$: erit $\dfrac{\zeta.\ T}{m-1.\ 360^\circ}$ intervallum *temporis* inter momentum *Elongationis maximæ* Planetæ inferioris & momentum *Stationis* proxime præcedentis vel subsequentis (num. 1. coll. p. 19. Cor. 3).

(*) Tunc (per EUCL. El. I. 32. & El. Trig. Pl.) innotescunt reliqui. Est nimirum Sin U : Sin Y :: n : 1; & Cotang $\frac{1}{2}$ Z : tang $\dfrac{U-Y}{2}$ :: $n+1 : n-1$.

§. XVI.

Planetis tempora periodica, respectu fixarum, in diebus & partibus decimalibus diei, hæc adsignat Ill. NEWTON (*a*)

b

$$My^2 + 2Ny + MX - \frac{dN}{dx} = 0;$$

$$2Mydy + y^2dM + 2Ndy + 2ydN + MdX + XdM - \frac{d^2N}{dx} = 0$$

$$\cancel{N = M}$$

$$dy + y^2\frac{dM}{2N} + \frac{XdM}{2N} = 0 = dy + y^2dx + Xdx$$

$$dx = \frac{dM}{2N}, \quad 0 = Nd^2M - dMdN$$

$$2Mydy + 2ydN + MdX - \frac{d^2N}{dx} = 0$$

$$\frac{dN}{dx} = \frac{2NdN}{dM}$$

$$d^2x=0; \quad \frac{d^3M}{2dx^2} - 2XdM + MdX = 0; \quad \frac{rdx}{2} - 2XdM + MdX = 0$$

$$dM = pdx, \quad dp = qdx, \quad dq = rdx$$

$$dx = \frac{dM}{2N}, \quad 2N = p, \quad qdx = q\frac{dM}{2N} = 2dN$$

$$d^2M = 0, \quad q = \frac{4NdN}{dM}, \quad \frac{rdx}{2} = \frac{4Nd^2N + 4dN^2}{dM}$$

$$dX = 2X\frac{dM}{M} - \frac{2Nd^2N}{MdM} - \frac{2dN^2}{MdM};$$

$$X = -2M^2\int\left(\frac{Nd^2N + dN^2}{M^3dM} + C\right) \quad dx = \frac{dM}{2N}; \quad \frac{m-2}{m-1}$$

$$M = ax^m, \quad N = \frac{max^{m-1}}{2}$$

$$dN = \overline{m-1}\,max^{m-2}dx, \quad d^2N = \overline{m-2}.\overline{m-1}\,max^{m-3}dx^2$$

$$Nd^2N = \overline{m-2}.\overline{m-1}\,m^2a^2x^{2m-4}dx^2 \quad dM = max^{m-1}dx$$

$$dN^2 = \overline{m-1}^2\,m^2a^2x^{2m-4}dx^2 \quad M^3 = a^3x^{3m}$$

$$+ \quad = (2m-3)(m-1)m^2a^2x^{2m-4}dx^2$$

$$\int \frac{\overline{2m-3}.\overline{m-1}\,m}{a^2}\,\frac{dx}{x^{2m+3}} = C - \frac{\overline{2m-3}.\overline{m-1}\,m}{\overline{2m+2}.a^2\,x^{2m+2}}$$

$$M^2 =$$

$\cancel{My^2 + 2Ny + M\int}$

$zy^2 + 2uy + zX - \frac{du}{dx} = 0$

$2zydy + y^2dz + 2udy + 2ydu + zdX + Xdz - \frac{d^2u}{dx} = 0$

$2udy + y^2dz + Xdz = 0 \quad 2zydy + 2ydu + zdX - \frac{d^2u}{dx}$

$dy + y^2\frac{dz}{2u} + \frac{Xdz}{2u} = 0 ; \quad 0 = dy + \frac{du}{z} + \frac{dX}{2y} - \frac{d^2u}{2zydx}$

$\frac{dz}{2u} = dx \quad 2u = \frac{dz}{dx}$

$M + \frac{N}{1 + \beta e^{\int N dx}} = y =$

$\frac{P + QN^{\int zdx}}{1 + \beta N^{\int zdx}} = y \quad d\left(\frac{z^n}{dz}\right) = \frac{nz^{n-1}dz}{} - \frac{z^{n-1}dz}{dz^2}$

$d\left(\frac{z}{dz}\right) = \frac{dz}{dz} - \frac{dz}{dz^2}$

$dy = \left[(dP + dQ + Qzdx)\,\beta N^{\int zdx}\right)\left(1 + \beta N^{\int zdx}\right)$

$-\left(P + QN^{\int zdx}\right)\left(\beta N^{\int zdx}\, zdx\right] : (1 + \beta N^{\int zdx})^2$

$N^{\alpha x^n} dx = \cancel{dz}$ $\quad \alpha x^n = y \quad N^{\alpha x} = y$

$\cancel{n\alpha x^{n-1}dx = \frac{dz}{dz}}$ $\quad n\alpha x^{n-1}dx = dy \quad \alpha dx = \frac{dy}{y}$

$N^{\alpha x^n} = z$ $\quad N^{\frac{\alpha}{x}} = z ; N^{\alpha x} dx = \frac{dz}{\alpha}$

$\alpha x^n = lz$ $\quad \frac{\alpha}{x} = lz$

$n\alpha x^{n-1}dx = \frac{dz}{z}$ $\quad -\frac{\alpha dx}{x^2} N^{\frac{\alpha}{x}} = dz$

$x = \frac{(lz)^{\frac{1}{n}}}{\alpha^{\frac{1}{n}}}$ $\quad N^{\frac{\alpha}{x}} dx = -\frac{x^2 dz}{\alpha} = \frac{\alpha dz}{lz^2}$

$n\alpha x^n dx = \frac{z^{-1}dz}{\alpha^{\frac{1}{n}}(lz)^{-\frac{1}{n}}}$ $\quad \int N^{\frac{\alpha}{x}} dx = A + \frac{\alpha z}{lz} - \alpha\int\frac{dz}{lz}$

$dx = \frac{z^{-1}dz}{n\alpha^{\frac{1}{n}}(lz)^{1-\frac{1}{n}}}$

$N^{\alpha x^n} dx = \frac{dz}{n\alpha^{\frac{1}{n}}(lz)^{1-\frac{1}{n}}}$

♄	♃	♂	♁
10759,275.	4332,514.	686,9785.	365,2565.

♀	☿
224,6176.	87,9692. (*)

(*a*) Princ L. III. Phæn. IV.

(*) Hæ revolutiones periodicæ ſatis commodæ mihi viſæ ſunt; vix autem differunt ab iis, quas D:nus DE LA CAILLE Aſtr. Sect. I. Part. I. Cap. II. Art. XIII. §. 170. edit. an. 1755. & alii (v. DE LA LANDE §. 852. p. 408. tab. Col. 2.) protulerunt.

§. XVII.

Pro SATURNO igitur, (vid. §. XV. & §. præc.) quatenus a Tellure noſtra ſpectatur, eſt $m : 1 :: 107592750 : 3652565$; adeoque $m = 29,456765$, Log. $m = 1.4691850$, $m - 1 = 28,456765$, $m + 1 = 30,456765$. Porro (§. X. p. 24.) $\frac{2}{3}$ log. $m =$ log. $n = 0.9794567$, ergo $n = 9,537987$, $n + 1 = 10,537987$, $n - 1 = 8,537987$; $m + n = 38,994752$ & $m - n = 19,918778$.

Jam pro *Stationibus* (§. XV. num. 1.)

$20 + \log.\overline{n+1} = 21.0227576$, cujus dimid. $= 10.5113788$

add. log. n $\underline{0.9794567}$ - - - ſubtr. $\underline{0.9794567}$

22.0022143 Log. Cot. $U = 9.5319221$.

dimid. $=$ L. Cot. $Y = 11.0011071$ $U = 108°,7959$

$Y = 5°,6962$, hinc $Z (= 180° - U - Y) = 65°,5079$

Eundem fere valorem, & quidem tam accurate, ut vix parte unius gradus $\frac{1}{10000}$ ab hoc differat, exhibent formulæ pro Sin. Z & Coſ Z. Porro igitur (§. XV. num. 3.)

$2Z = 131,0158$ cujus $\underline{\text{Log.}} = 2.1173237$

ſubtr. Log $\overline{m - 1}$ $\underline{1.4541855}$

Log $4°,6040 = 0.6631382$

Log.

$\text{Log. } 360^\circ = 2.5563025$ $\qquad 2Y = 11^\circ,3924$

$\text{subt } L\,\overline{m-1} \quad 1.4541855$ $\qquad \text{subt. } \frac{2Z}{m-1} \quad 4,6040$

$\text{Log} \frac{360^\circ}{m-1} = 1.1021170$ $\qquad 6^\circ,7884 = $ *Arc. Retr.*

$\frac{360^\circ}{m-1} = 12^\circ,6508$ (*) - - add. $12,6508$

$19^\circ,4392$, *arcus* motu *Directo* descriptus. Deinde (vid. §. XV. 2), quia T = 10759,275., est Log. T = 4.0317830 cui addatur supra

$\text{inventus Log } \frac{2Z}{m-1} \quad 0.6631382$

4.6949212

$\text{subtr. Log } 360^\circ \quad 2.5563025$

2.1386187 cui respondent $137^d,6001 = 137^d,14^h,24'$ &c. = *Durationi Retrogradationis*. Denique, ob $\text{Log } \frac{T}{m-1} = 2.5775975$, *Periodus Phænomenorum* (§. XV. 4) $= 378^d,0920$, unde subtracto tempore retrogradationis $= 137^d,6001$, fit *Duratio Progressionis* $= 240^d,4919 = 240^d,11^h,48'$. Ad determinandam durationem progressionis adhiberi etiam possunt Formulæ §. XV. 2. quæ prorsus eundem dant valorem.

Ab Oppositione igitur ad conjunctionem & contra (§. XV. 4.) pervenit ♄ tempore $189^d,0460$ & interea percurrit
justo major motu vero arcum $6^\circ,3254$, Tellus vero arcum $186^\circ,3254$,
erit enim quia $\text{Log } \frac{m.\,180^\circ}{m-1} = 2.2702720$. Adeoque absoluto tempo-
$= m.6^\circ,3254$ re Phænomenorum emensa est Tellus arcum $372^\circ,6508$.

(*) Hic est arcus, quem ♄ tempore Phænomenorum motu vero descripsit, (§. VI. β) p. 11. cfr. §. XV. 4.).

§. XVIII.

$y = \frac{u}{1+\alpha N^{\int z dx}}$

$dy = \frac{(1+\alpha N^{\int z dx}) du - \alpha N^{\int z dx} u z dx)}{(1+\alpha N^{\int z dx})^2}$

$+ y^2 dx = + \frac{u^2 dx}{(1+\alpha N^{\int z dx})^2}$

$+ X dx = X dx$

$0 = (1+\alpha N^{\int z dx}) du + dx [u^2 - \alpha N^{\int z dx} u z + X.(1+\alpha N^{\int z dx})^2]$

$du + \alpha du N^{\int z dx} + \alpha^2 X dx . N^{2 \int z dx} = 0$
$+ u^2 dx \quad - \alpha u z dx N^{\int z dx}$
$+ X dx \quad + 2\alpha X dx . N^{\int z dx}$

$du (1+\alpha N^{\int z dx}) - \alpha u z dx N^{\int z dx} +$

$\overline{1+\alpha N^{\int z dx}} = v$

$v du - u dv + 2.\overline{v-1} X dx = 0$

$u^2 + X + \overline{v-1}^2 X = 0$

$y = \frac{u}{p}, \quad dy = \frac{p du - u dp}{p^2}$

$y^2 = \frac{u^2}{p^2}$

$p du - u dp + u^2 dx + p^2 X dx = 0$

$p du + p^2 X dx = 0$

$p = - \frac{du}{X dx}, \quad dp + u dx = 0$

$u = - \frac{dp}{dx}, \; du = \frac{d^2 p}{dx}$

$d^2 p = p X dx \qquad \frac{dp}{p} = dz, \; \frac{d^2 p}{p} = d^2 z + dz^2$

$y = - \frac{dp}{p\, dx} \qquad d^2 z + dz^2 = X dx^2$

$d^2y + dy^2 = \alpha x^4 dx^2$

$d^3y + 2dyd^2y = \alpha dx^3$

$d^4y + 2dyd^3y + 2d^2y^2 = 0$

$\cancel{N^{\lambda y}(d^3y + 2dyd^2y) =}$

$d^2y = pdy^2$

$d^3y = dpdy^2 + 2p^2dy^3$

$d^4y = d^2pdy^2 + 6pdpdy^3 + 6p^3dy^4$

$d^2p + 6pdpdy + 6p^3dy^2$
$+2dpdy + 4p^2dy^2$
$+2$

$d^2p + (6p+2)dpdy + (p^3+p^2)6dy^2 = 0$

$\cancel{dp = udy \cdot + d^2p = dudy + updy^2}$

$\cancel{dy = udp, \quad pu^2dp = dudp + ud^2p}$

$\cancel{d^2p = pudp^2 - \frac{dudp}{u}}$

$\cancel{\frac{dudp}{u} = (7p+2)udp^2 + (p^3+p^2)6u^2dp}$

$dp = udy, \quad d^2p = dudy + updy^2$

$= dydu + pdpdy$

$dy = \frac{dp}{u}$

$du + (6p+2)dp + (p^3+p^2)\frac{6dp}{u} = 0$

$udu + (7p+2)udp + \overline{6p+1}\,p^2dp = 0$

$N^{\alpha lx} = z$

$7pdp + 2dp = dz$

$\alpha lx = \cancel{\frac{dz}{z}} = lz$

$\frac{7}{2}p^2 + 2p + \alpha = z$

$x^\alpha = z$

$7\beta p + 2\beta dp = \beta\frac{dz}{dp}$

$\frac{\alpha dx}{x}$

$\frac{47}{2}\beta p^3 + 21\beta p^2dp + 4\beta$

§. XVIII.

Quatenus motus geocentricus spatio unius diei seu 24^h æquabilis quam proxime censeri potest, igitur, posita $c = \left(\frac{360}{T} = \right) \frac{1296000}{T}$ sec. ubi T, ut supra, denotat dies, arcus tunc descriptus velocitatem adparentem designabit. Sic in Syzygiis (§. XV. 5.)

Log 1296000 =	6.1126050		
subtr. Log T	4.0317830	Log $\overline{m-n}$ =	1.2992627
Log c =	2.0808220	- - - add.	2.0808220
add. Log $\overline{m+n}$	1.5910062		3.3800847
	3.6718282	subtr. Log $\overline{n-1}$	0.9313554
subtr. Log $\overline{n+1}$	1.0227576	Log $-v$ =	2.4487293
Log v =	2.6490706		
v =	445'',7287	$-v$ =	281'',0149

Ergo ♄, cum velocissime progreditur, 24^h spatio videtur emetiri arcum 7', 26'', celerrima autem retrogradatione arcum 4', 41'', quam proxime.

§. XIX.

Quæratur (§. XV. num. 7.) mora temporis, quo motus adparens (spatio 24^h) non fit > 1'; adeoque hic erit $c = \frac{21600}{T}$, & Log $c = 0.3026707$, Log $\frac{v}{c}$ = Log $b = -$ Log $c = \bar{1}.6973293$. Proinde $b = 0{,}498115$, $m + b = 29{,}95488$ $(= 864 \times 0{,}03467)$, $m - b = 28{,}95865$, $1 + b = 1{,}498115$, $1 - b = 0{,}501885$ $(= 855 \times 0{,}000587)$, $m + 1 + 2b = 31{,}452995$, $m + 1 - 2b = 29{,}460535$. Ergo (§. XV. 8.).

$\text{Log}.\overline{m - b} = 1.4617783$ $\text{Log}\,\overline{1 - b} = \bar{1}.7006042$
ſubtr. Log. n 0.9794567 - - - add. 0.9794567

$\text{Log}\,A = 0.4823216$ $\text{Log}\,B = 0.6800609$
$A = 3,036138.$ $B = 4,786972.$

Adeoque $A + B = 7,82311$, $10 + \text{Log}\,\overline{A + B} = 10.8933794$
ſubtr. $\text{Log}\,\overline{m + 1 - 2b}$ 1.4692406

$\text{Log Coſ}\,Z = 9.4241388$
$Z = 74^\circ,6006$. Porro

$\text{Log}\,\overline{m + b} = 1.4764675$ $\text{Log}\,\overline{1 + b} = 0.1755451$
ſubtr. Log n 0.9794567 - - add. 0.9794567

$\text{Log}\,\mathfrak{A} = 0.4970108$ $\text{Log}\,\mathfrak{B} = 1.1550018$
$\mathfrak{A} = 3,140587$ $\mathfrak{B} = 14,289.$

$\mathfrak{A} + \mathfrak{B} = 17,429587$, $10 + \text{L}.\overline{\mathfrak{A} + \mathfrak{B}} = 11.2412871$
ſubtr. $\text{Log}\,\overline{m + 1 + 2b}$ 1.4976619

$\text{Log Coſ}\,\mathfrak{Z} = 9.7436252$. $\mathfrak{Z} = 56^\circ,3480$.

Ergo $Z - \mathfrak{Z} = 18,2526$ cujus $\text{Log} = 1.2613248$
add. $\text{Log}\,\dfrac{T}{m - 1}$ 2.5775975

3.8389223
ſubtr. Log 360° 2.5563025

$\text{Log}\,\dfrac{Z - \mathfrak{Z}}{m - 1.\,360^\circ}.T = 1.2826198.$ adeoque

$\dfrac{Z - \mathfrak{Z}}{m - 1.\,360^\circ}.T = 19^d,1699$ ($= 19^d,4^h,4'$) mora illa (§. XV. 8), quæ quidem erit duratio ſtationis Saturni, ſi Planeta ſtare cenſendus ſit, dum motus ejus eſt adeo lentus, non autem ſtare, ſi velocius moveri videatur.

Poſſent etiam quærendo U & Y tam pro $-b$ quam $+b$ reperiri Z, $\mathfrak{Z}$ &c. (§. XV. 7. 8); ſed his immorari nolo.

§. XX.

$$d^4y + 2dyd^3y + 2d^2y = 0$$

$$N^{\delta y}(d^3y + \alpha dyd^2y) = Adx^3$$

$$d^4y + \alpha dyd^3y + \alpha d^2y^2$$
$$+\delta = 0$$

$$d^3y + 2dyd^2y = adx^3$$

$$N^{\delta y} dy^n (d^2y + \beta dy^2) = \alpha\beta dx^{2+n}$$

$\cancel{N}$ $d^3y + 2\beta$

$\cancel{N^{\delta y}(dy + \alpha x^m dx)} =$ $\quad y = N^{\int v dx}$

$$dy = v dx N^{\int v dx}$$

$$dy = \alpha x^m dx \qquad d^2y = dv dx N^{\int v dx}$$
$$+ v^2 dx^2 N^{\int v dx}$$

$$d^2y = m\alpha x^{m-1} dx^2$$

$$d^2y + ayx^n dx^2 = 0$$

$$x = N^{\int v^m z^r dz}, \ \& \ y = N^{\int v dz} \ ; d^2z = 0$$

$$dy = v dz N^{\int v dz}, \ d^2y = (dv dz + v^2 dz^2) N^{\int v dz}$$

$$dv + v^2 dz + \alpha N^{n \int v^m z^r dz} v^m z^r dz = 0$$

$$m = 2 ; -\frac{dv}{v^2} = dz\left(1 + \frac{a z^r}{v} N^{n \int \frac{z^r dz}{v^2}}\right);$$

$$y = u^m z, \ x = u^r z^\delta$$

$$dy = u^m dz + m u^{m-1} z du$$

$$dx = \delta u^r z^{\delta-1} dz + r u^{r-1} z^\delta du$$

$$y^2 dx = \delta u^{r+2m} z^{\delta+1} dz + r u^{r+2m-1} z^{\delta+2} du$$

$$\alpha x^n dx = \alpha\delta u^{\overline{n+1}r} z^{\overline{n+1}\delta-1} dz + \alpha r u^{\overline{n+1}r-1} z^{\overline{n+1}\delta} du$$

$$+ dy = u^m dz + m u^{m-1} z du \qquad \cancel{\frac{2n+2}{n+2}}$$

$$m = \overline{n+1}\cdot\overline{\delta+r} - 1 = r + \delta + 2m + 1 \qquad \cancel{\delta + r = \frac{2}{n+2}}$$

$$m = \frac{n}{n+2}$$

$$\delta + r = \frac{m+1}{n+1} = -1-m \qquad 1 + m = \frac{2n+2}{n+2}$$

$$m + r = n + r - \overline{n+1}m \qquad \cancel{1 - m = \frac{2}{n+2}} \qquad r + \delta = -\frac{2n+2}{n+2}$$

$$m\cdot\overline{n+2} = n$$

$dy + y^2 dx + \alpha dx = 0 =$

$2\int N dx + \ell \frac{y + M - N}{y + M + N} = C.$

$2N dx$

$d^2 y + dy^2 + a dx^3 = 0$

$q + p^2 + a = 0$

$dp = q dx = \frac{q dy}{p}$

$q = \frac{p dp}{dy} \qquad p dp + \overline{p^2 + a}\, dy = 0$

$\frac{p dp}{p^2 + a} + dy = 0$

$\frac{2 p dp}{p^2 + a} + 2 dy = 0$

$2y + \ell \overline{p^2 + a} = \ell C$

$N^{2y}(p^2 + a) = C$

$\frac{d^2 y}{y} + \alpha x dx^2 = 0 \qquad dx = \frac{dy}{p} = \frac{dp}{q} = \frac{dq}{r}$

$\frac{d^3 y}{y} - \frac{dy d^2 y}{y^2} + a dx^3 = 0 \quad \frac{r}{y} - \frac{pq}{y^2} + a = 0$

$ry - pq + ay^2 = 0 \ , \ r = p\frac{dq}{dy} \ , \ q = p\frac{dp}{dy}$

$dq = p\frac{d^2 p}{dy} + \frac{dp^2}{dy} \ ; \ d^2 y = 0 \quad dp = u dy + y du$

$r = p^2 \frac{d^2 p}{dy^2} + p\frac{dp^2}{dy^2} \qquad \frac{dp}{dy} = u + $

$p^2 y \frac{d^2 p}{dy^2} + py\frac{dp^2}{dy^2} - p^2 \frac{dp}{dy} + ay^2 = 0$

$dp =$

$p = uy \ , \ \frac{d^2 p}{dy^2} = \frac{v}{y} \ ;$

$p^2 v + \frac{p^2 dp^2}{u dy^2} - \frac{p^2 dp}{dy} + \frac{ap^2}{u^2} = 0$

$y = \frac{p}{u} = \frac{d^2 p}{v dy^2} \qquad v + \frac{dp^2}{u dy^2} - \frac{dp}{dy} + \frac{a}{u^2} = 0$

$\frac{v}{y} = \frac{vu}{p}$

§. XX.

Determinetur quoque (§. XV. 6) velocitas adparens ♄ pro quocunque ejus loco dato. Sit e. g. $U = 60°$, adeoque $\text{Log Sin } U$ ($= \text{Log Sin } 60°$) $= 9.9375306$, ex quo (quia $\text{Sin } Y = \frac{\text{Sin } U}{n}$) detracto $\text{Log } n = 0.9794567$, fit $\text{log Sin } Y = 8.9580739$, $Y = 5°,2095$ & $Z = (180° - U - Y =)$ $114°,7905$ cujus log Sin $= 9.9580127$. Jam

$$\begin{array}{llll}
 & & \text{Log. Sin } U = & 9.9375306 \\
 & & \text{Log. Cof } U & 9.6989700 \quad \text{add.} \\
10 + \text{Log Sin } Z = & 19.9580127 & \text{Log } m - 1 & 1.4541855 \\
\text{add. Log } n & 0.9794567 & & 21.0906861 \\
 & 20.9374694 & \text{- - fubtr.} & 20.9374694 \\
 & & \text{Log } \gamma = & 0.1532167,
\end{array}$$

$\gamma = 1,423039$, $1 + \gamma = 2,423039$ cujus Log $= 0.3843604$
add. (§. XVIII.) Log c $\quad 2.0808220$
Log $v = 2.4651824$.

Ergo velocitas ♄ adparens (ubi $U = 60°$) vel $v = 4',51'',8653$.

§. XXI.

Pro JOVE eft (§§. XV. XVI.) $m = 11,861566$, Log. $m = 1.0741420$, $\frac{2}{3}$ log. $m =$ log. $n = 0.7160947$ & $n = 5,201094$, $n + 1 = 6,201094$, $n - 1 = 4,201094$, $m - 1 = 10,861566$, $m + 1 = 12,861566$, $m + n = 17,06266$, $m - n = 6,660472$. Ergo pro *Stationibus* (§. XV. n. 1).

$$\begin{array}{llll}
20 + \log \overline{n+1} = & 20.7924683 & \text{cujus dimid.} = & 10.3962341 \\
\text{add. Log } n & 0.7160947 & \text{- - - fubtr.} & 0.7160947 \\
 & 21.5085630 & \text{Log Cot. } U = & 9.6801394 \\
\text{dim.} = \text{L. Cot. } Y = & 10.7542815 & & U = 115°,5843.
\end{array}$$

$Y = 9°,9865$, ergo $Z = 54°,4292$.

Adhibitis formulis pro Sin Z & Cof Z reperitur Z adcurate ut fupra. Hinc porro (§. XV. n. 3)

$2Z = 108^\circ,8584$, ejusque Log $= 2.0368620$

subtr. Log $\overline{m-1}$ $\quad 1.0358924$

Log. $10^\circ,0224 = 1.0009696$

Log. $360^\circ = 2.5563025$ $\quad 2Y = 19^\circ,9730$

subt. L. $\overline{m-1}$ $\quad 1.0358924$ subt. $\frac{2Z}{m-1}$ $\quad 10,0224$

1.5204101 $\quad 9^\circ,9506 =$ *Arc. Retr.*

$\frac{360^\circ}{m-1} = 33^\circ,1444$ - - add. $33,1444$

$43^\circ,0950 =$ *Arc. Dir.*

Porro (§. §. XVI. XV. 2) Log. T $= 3.6367400$

add. Log. $\frac{2Z}{m-1}$ $\quad 1.0009696$

4.6377096

subtr. log 360° $\quad 2.5563025$

Log. $\frac{Z.T}{m-1.\,180^\circ} = 2.0814071$, adeoque *Duratio Retrogradationis* $= 120^d,6166 = 120^d, 14^h, 48'$, quæ detracta a *Periodo Phænomenorum* (§. XV. 4) $= 398,8849$ Facit *Durationem Progressionis* $= 278^d,2683$. Subducto calculo secundum §. XV. n. 2 fit Duratio Directionis ut supra.

Inter oppositionem & subsequentem conjunctionem intercedit Tempus $199^d,4424$, quo Jupiter percurrit arcum $16^\circ,5722$ & Tellus (ob Log $\frac{m.\,180^\circ}{m-1} = 2.2935221$) $196^\circ,5722$. Periodo autem Phænomenorum percurritur a Jove arcus $33^\circ,1444$ & a Tellure arcus $393^\circ,1444$.

§. XXII.

In Syzygiis (vid. §. XV. 5. §. XVIII.)

Log.

$\int P dx + \mathrm{l}\frac{\cdot}{y+\psi} = C.$

$$P dx + \frac{\frac{dy+d\varphi}{y+\varphi}}{y+\psi} - \frac{dy+d\psi}{y+\psi} = 0$$

$$\cancel{Py^2dy + \overline{\varphi+\psi}\,Pydy + P\varphi\psi dy}$$

$$\begin{array}{l} ydy + yd\varphi + \psi dy + \psi d\varphi + y^2 P dx \\ -ydy - yd\psi - \varphi dy - \varphi d\psi \; * \\ \quad + y\varphi P dx \qquad + \varphi\psi P dx \\ \quad + y\psi P dx \end{array}$$

$$dy + y^2 dx + X dx = 0$$

$$d\varphi - d\psi + \overline{\varphi+\psi}\,P dx = 0$$

$$dy + y^2\frac{P}{\psi-\varphi}dx + \frac{\psi d\varphi - \varphi d\psi + \varphi\psi P dx}{\psi-\varphi} = 0$$

$P = \psi - \varphi$; $\quad \psi + \varphi = \frac{dP}{Pdx}$

$\frac{d\psi - d\varphi}{\psi - \varphi} = \overline{\psi+\varphi}\,dx \quad \overline{\psi+\varphi}^2 = \frac{dP^2}{P^2dx^2} = \psi^2 + \varphi^2 + 2\varphi\psi$

$\varphi\psi = \frac{dP^2}{4P^2dx^2} - \frac{1}{4}P^2 \quad \overline{\psi-\varphi}^2 = P^2 = \psi^2 + \varphi^2 - 2\varphi\psi$

$\frac{\psi d\varphi - \varphi d\psi}{\psi^2} = \frac{d\omega}{\psi^2} = d.\frac{\varphi}{\psi}$; $\quad \psi = \frac{dP}{2Pdx} + \frac{1}{2}P$

$\qquad \varphi = \frac{dP}{2Pdx} - \frac{1}{2}P$

$d\omega = \psi^2 . d.\frac{\varphi}{\psi}$;

$$4d\omega = \left(\frac{dP}{\cancel{4}Pdx} + \cancel{1}P\right)\left(\frac{d^2P}{\cancel{2}Pdx} - \frac{dP^2}{\cancel{2}P^2dx} - \cancel{1}dP\right)$$

$$- \left(\frac{dP}{\cancel{4}Pdx} - \cancel{1}P\right)\left(\frac{d^2P}{\cancel{2}Pdx} - \frac{dP^2}{\cancel{2}P^2dx} + \cancel{1}dP\right)$$

$$4PXdx = \cancel{\frac{dPd^2P}{P^2dx^2}} - \cancel{\frac{dP^3}{P^3dx^2}} - \frac{\cancel{4}dP^2}{Pdx} + \frac{2d^2P}{dx} - \cancel{PdP}$$

$$+ \frac{dP^2}{Pdx} \qquad - P^3dx$$

$$\frac{d^2P}{Pdx} - \frac{dP^2}{P^2dx} - \frac{1}{2}P^2dx = 2Xdx \quad \Big| \quad \frac{dP}{P} = dz$$

$P = N^z$; $\quad d^2z - \frac{1}{2}N^{2z}dx^2 = 2Xdx^2$

$$\frac{d^2z - 2X dx^2}{\frac{1}{2}dx^2} \doteq N^{2z}$$

$$d^3z - 2dX dx^2 = 2dz\, d^2z - 4X dz\, dx^2$$

$$d^3z - 2dz\, d^2z + 4X dz\, dx^2 - 2dX dx^2 = 0$$

$$r - 2pq + 4pX - 2X' = 0$$

$$d^2z = p\, dz^n dx^{2-n}$$

$$dp\, dz^n dx^{2-n} + np^2 dz^{2n-1} dx^{4-2n} - 2p\, dz^{n+1} dx^{2-n}$$

$$+ \&c = 0 \qquad \cancel{n} = 0,\ \cancel{p=}$$

$$dp - 2p\, dz + 4X dz - 2dX = 0$$

$$p = 2X,$$

$$y^2x\frac{d^2y}{dx^2} + yx\frac{dy^2}{dx^2} - y^2\frac{dy}{dx} + ax^2 = 0$$

$$y^2xq + yxp^2 - y^2p + ax^2 = 0$$

$$y = ux,\ q = \frac{v}{x}$$

$$vu^2x^2 + ux^2p^2 - u^2x^2p + ax^2 = 0$$

$$vu^2 + up^2 - u^2p + a = 0\ ,\quad v = \overline{p-u}\frac{dp}{du}$$

$$\overline{p-u}\frac{u^2dp}{du} + up^2 - u^2p + a = 0$$

$$\overline{p-u}\,dp + p^2\frac{du}{u} - p\,du + a\frac{du}{u^2} = 0$$

$$\frac{p}{u} = z\ ,\ u\,dp + p\,du = ds$$

$$pu = s \qquad p\,dp = \tfrac{1}{2}s\,dz + \tfrac{1}{2}z\,ds$$

$$p^2 = sz \qquad d^2y + \frac{dy^2}{y} - \frac{dy\,dx}{x} + \frac{ax\,dx^2}{y^2} = 0$$

$$\frac{s}{z} = u^2$$

Log 1296000 = 6.1126050
ſubtr. log. T 3.6367400 Log. $\overline{m-n}$ = 0.8235050
Log c = 2.4758650 - - - add. 2.4758650
add. Log $\overline{m+n}$ 1.2320467 3.2993700
3.7079117 ſubtr. log $\overline{n-1}$ 0.6233624
ſubtr. log. $\overline{n+1}$ 0.7924683 Log $-v$ = 2.6760076
Log v = 2.9154434 $-v$ = 474″,2503
v = 823″,0826

Adeoque maxima velocitas directa eſt 13′,43″, &c. maxima autem retrograda 7′,54″ &c.

§. XXIII.

Pro inveniendo intervallo temporis, quo Jupiter Stationarius adpareat, (§. XV. n. 7 cfr. §. XIX.), eſt Log c = 0.6977137 & Log $\frac{v}{c}$ = Log b = $-$ log c = $\overline{1}$.3022863.

Ergo b = 0,200579, $m+b$ = 12,062145, $m-b$ = 11,660987, $1+b$ = 1,200579, $1-b$ = 0,799421, $m+1+2b$ = 13,262724, $m+1-2b$ = 12,460408. Adeoque (§. XV. 8)

Log $\overline{m-b}$ = 1.0667352 log. $\overline{1-b}$ = 1.9027755
ſubt. log n = 0.7160947 - - add. 0.7160947

Log A = 0.3506405 Log B = 0.6188702
A = 2, 242025 B = 4, 157863

A+B = 6,399888, 10+L. $\overline{A+B}$ = 10.8061723
ſubtr. Log $\overline{m+1-2b}$ 1.0955322
Log Coſ Z = 9.7106401, Z = 59°,0949

Log $\overline{m+b}$ = 1.0814246 Log $\overline{1+b}$ = 0.0793907
ſubt. Log n 0.7160947 - - add. 0.7160947

Log 𝔄 = 0.3653299 Log 𝔅 = 0.7954854
𝔄 = 2, 319156 𝔅 = 6, 244324

$\mathfrak{A}+\mathfrak{B}=8,56548$, $10+L.\overline{\mathfrak{A}+\mathfrak{B}}=10.9326503$
subtr. Log. $\overline{m+1+2b}$ $\quad 1.1226327$
Log Cof Z $=9.8100176$, $\mathfrak{Z}=49°,7833$
ergo $Z-\mathfrak{Z}=9°,3116$ cujus log $=0.9690243$
add. Log $\frac{T}{m-1}$ $\quad 2.6008476$

3.5698719
subtr log 360° $\quad 2.5563025$

1.0135694, cui respondent $10^d,3174$ h. e. intervallo $10^d,7^h,37'$ quam proxime motus ♃:is adparens (spatio 24^h) non excedit $1'$.

§. XXIV.

Velocitas Jovis adparens ubi $U=60°$ sic reperitur (§. XV. 6):
Log Sin U $=9.9375306$
subtr. Log n $\quad 0.7160947$
Log Sin Y $=9.2214359$
$Y=9°,5849$ $\qquad$ log Sin U $=9.9375306$
ergo $Z=110,4151$ $\qquad$ log Cof U $\quad 9.6989700$ add.
$10+$log Sin Z $=19.9718276$ $\qquad$ log $\overline{m-1}$ $\quad 1.0358924$
add. log n $\quad 0.7160947$ $\qquad$ 20.6723930
20.6879223 - - subtr. $\quad 20.6879223$
Log $\gamma=\bar{1}.9844707$ &
$\gamma=0,964874$, $1+\gamma=1,964874$ ejusque log $=0.2933347$
add. (§. XXII.) log c $\quad 2.4758650$
Log. $v=2.7691997$
adeoque hoc in casu $v=9',47'',7595$.

§. XXV.

Pro MARTE (§. §. XV. XVI.) $m=1,880811$, Log $m=0.2743451$, $\frac{2}{3}$ log $m=$ log $n=0.1828967$ unde $n=1,523690$,

$n+1$

$$\frac{d^2y}{y} + A dx^2 = 0, \quad y = a N^{\ldots}$$

$$ly = a + A\int dx \int X dx$$

$$\frac{dy}{y} = A dx \int X dx, \quad \frac{d^2y}{y} = A X dx^2 + A^2 dx^2 \left(\int X dx\right)^2.$$

$$y = N^{A\int dx \int X dx} + \alpha N^{-A\int dx \int X dx}$$

$$dy = A dx \int X dx \left(N^{A\int dx \int X dx} - \alpha N^{-A\int dx \int X dx}\right)$$

$$d^2y = A X dx^2 \left(N^{A\int dx \int X dx} - \alpha N^{-A\int dx \int X dx}\right)$$
$$+ A^2 dx^2 \left(\int X dx\right)^2 \left(N^{A\int} + \alpha N^{-A\int}\right)$$

$$\frac{d^2y}{y} = A^2 dx^2 \left(\int X dx\right)^2$$

$$\int X dx = X^2 \quad \frac{dX}{X^2} = X dx;$$

$$N^{\int z dx} = N^{-\int \varphi dx} \left(\alpha N^{\int \psi dx} + \beta N^{-\int \psi dx}\right)$$

$$\int z dx = -\int \varphi dx + L\left(\alpha N^{\int \psi dx} + \beta N^{-\int \psi dx}\right)$$

$$\frac{z + \varphi}{\psi} = \frac{\alpha N^{\int \psi dx} - \beta N^{-\int \psi dx}}{\alpha N^{\int \psi dx} + \beta N^{-\int \psi dx}}$$

$$y\,dy + y\,dx + ax^n dx = 0; \quad y = x^n z$$

$$dy = x^n dz + n x^{n-1} z dx$$

$$y = z + a x^n, \quad dy = dz + n a x^{n-1} dx$$

$$z dz + a x^n dz +$$

$$y = N^z, \quad y\,dy = N^{2z} dz + N^z dx$$

$$y d^2y + dy^2 + dy dx + n a x^{n-1} dx^2 = 0$$

$$y + \frac{dy^2}{d^2y} + \frac{dy dx}{d^2y} + \frac{n a x^{n-1} dx^2}{d^2y} = 0$$

$$3dy - \frac{dy^2 d^3y}{d^2y^2} - \frac{dy dx d^3y}{d^2y^2} - \frac{n a x^{n-1} dx^2 d^3y}{d^2y^2} + dx +$$

$ydy + ydx + ax^n dx = 0$

$y = u^p z^q, \; x = u^m z^n, \; dx = mu^{m-1} z^n du + nu^m z^{n-1} dz$

$ydy = pu^{2p-1} z^{2q} du + qu^{2p} z^{2q-1} dz$

$ydx = mu^{p+m-1} z^{q+n} du + nu^{p+m} z^{q+n-1} dz$

$+ax^r dx = mau^{\overline{r+1}m-1} z^{\overline{r+1}n} du + nau^{\overline{r+1}m} z^{\overline{r+1}n-1} dz$

$2p-1 \quad p = \frac{\overline{r+1}m}{2} = \frac{m}{2}$

$p=1, q=0, m=0$

$ydy + ydx + ax^n dx = 0$

$x = y^r z, \quad dx = y^r dz + ry^{r-1} z dy$

$ydy + ry^r z dy + y^{r+1} dz + ay^{\overline{n+1}r} z^n dz + ray^{\overline{n+1}r-1} z^{\overline{n+1}} dy$

$dy(1 + ry^{r-1} z + ray^{\overline{n+1}r-2} z^{\overline{n+1}}) +$

$+ dz(y^r + ay^{\overline{n+1}r-1} z^n) = 0$

$y^s + \beta y^\pi z^\mu$

$y^{r+s} + ay^{\overline{n+1}r+s-1} z^{n+s} + a\beta y^{\overline{n+1}r+\pi-1} z^{n+\mu}$

$+ ay^{r+\pi} z^{\mu+r}$

$+ \beta y$

$r+s=0, \quad \pi = -1; \quad nr-1 = r-1$

$n=1$

$r=0, \; n=1, \; dz\left(1 + \frac{az}{y}\right) + dy = 0$

$ydz + azdz + dy$

$dy = zdp, \; d^2y = dzdp + zd^2p = pz^n dp^n dx^{2-n}$

$d^2p = \frac{-dzdp}{z} + pz^{n-1} dp^n dx^{2-n}$

$\frac{dz}{z} + pz^{n-1} dp^{n-1} dx^{2-n} + 3npz^{n-1} dp^{n-1} dx^{2-n} + azdp$

$+ \overline{2n-1} np^3 z^{2n-2} dp^{2n-3} dx^{4-2n} + \overline{na+b} p^2 z^n dp^{n-1} dx^{2-n}$

$+ cpz^2 dp + ez^4 dp^{3-n} dx^{n-2} = 0$

$n+1 = 2,523690$ $(= 27 \times 0,09347)$, $n-1 = 0,523690$, $m+1 = 2,880811$, $m-1 = 0,880811$, $m+n = 3,404501$, $m-n = 0,357121$. Adeoque pro *Stationibus*

$20 + \log \overline{n+1} = 20.4020360$ cujus dimid. $= 10.2010180$
add. $\log n$ 0.1828967 - - subtr. 0.1828967
20.5849327 Log Cot U $= 10.0181213$
dim. $=$ log Cot Y $= 10.2924663$ U $= 136^\circ,1950$,
Y $= 27^\circ,0196$. ergo Z $= 16^\circ,7854$
2Z $= 33^\circ,5708$ cujus Log $= 1.5259617$
subtr. $\log m-1$ $\bar{1}.9448827$
Log $38^\circ,1135 = 1.5810790$.
Log $360^\circ = 2.5563025$ 2Y $= 54^\circ,0392$
subt. $\log m-1$ $\bar{1}.9448827$ subt. $\frac{2Z}{m-1}$ $38,1135$
2.6114198 $15^\circ,9257$, *Arc. Retr.*
$\frac{360^\circ}{m-1} = 408^\circ,7143$ - - add. $408,7143$
$424^\circ,6400$, *Arc. Dir.*

Deinde (§. §. XVI. XV. *n.* 2.) log. T $= 2.8369431$
add. Log $\frac{2Z}{m-1}$ 1.5810790
4.4180221
subtr. Log 360° 2.5563025
Log $\frac{Z.\,T}{m-1.\,180^\circ} = 1.8617196$ cui respondent $72^d,7310 = 72^d,17^h,33'$, *Durationem retrogradationis* indicantes. Jam vero Log $\frac{T}{m-1} = 2.8920604$ & hinc *Periodus Phænomenorum* $= 779^d,9386$ adeoque *Duratio Progressionis* $= 707^d,2076 = 707^d,4^h,59'$ quam proxime. Ab oppositione ad conjunctionem labitur tempus $389^d,9693$, quo

quo Mars emensus est motu vero orbitæ suæ arcum 204°,3571, Tellus autem, quia $\frac{m.180°}{m-1} = 2.5847349$, arcum 384°,3571. Duplicando hos arcus inveniuntur quoque arcus periodo Phæn. a Marte & Tellure descripti (cfr. §. XV. n. 4).

§. XXVI.

Celeritates adparentes in Syzygiis (§. XV. 5. cfr. §. XVIII.) calculo sequenti eliciuntur.

Log 1296000 =	6.1126050		
subtr. Log. T	2.8369431	Log $\overline{m-n}$ =	$\bar{1}$.5528153
Log c =	3.2756619	- - - add.	3.2756619
add. Log $\overline{m+n}$	0.5320534		2.8284772
	3.8077153	subtr. Log $\overline{n-1}$	$\bar{1}$.7190743
subt. Log $\overline{n+1}$	0.4020360	Log $-v$ =	3.1094029
Log v =	3.4056793	$-v$ =	1286″,4797
v =	2544″,9502		

Ergo motu directo velocissimo (spatio 24h) emetitur ♂ arcum 42′, 25″, celerrima autem retrogradatione arcum 21′, 26″ quam proxime.

§. XXVII.

Si etiam pro Marte invenire oporteat moram *Stationis* eo scilicet sensu, quo supra (§. XV. 7. 8. cfr. §. XIX.) diximus, erit Log c = 1.4975106 & log $\frac{v}{c}$ seu log b = $-$ log c = $\bar{2}$.5024894; ideoque b = 0,031805, $m+b$ = 1,912616, $m-b$ = 1,849006, $1+b$ = 1,031805 (= 135 × 0,007643), $1-b$ = 0,968195, $m+1+2b$ = 2,944421, $m+1-2b$ = 2,817201. Jam igitur

Log.

$dy\,d^3y + a\,d^2y^2 + b\,dy^2d^2y + c\,dy^4 = 0$, $dy = q\,dx$

$d^2y = p\,dy^2 = pq^2dx^2 = dq\,dx$, $dx = \frac{dq}{pq^2}$

$dy = \frac{dq}{pq}$, $d^2y = \frac{dq^2}{pq^2}$

$dq = pq^2dx$, $d^2q = (q^2dp + 2pq\,dq)\frac{dq}{pq^2} = \frac{dp}{p} + \frac{2dq}{q}$

$d^3y = (dp + 2p\,d^2y)\,dy^2 = \left(dp + 2p\frac{dq}{q}\right)\frac{dq^2}{p^2q^2}$

$$dy\,d^3y = \frac{dp\,dq^3}{p^3q^3} + \frac{2dq^4}{p^2q^4} + \frac{b\,dq^4}{p^3q^4} + \frac{c\,dq^4}{p^4q^4} = 0$$
$$+ a\frac{dq^4}{p^2q^4}$$

$$p\,dp + \frac{dq}{q}\left(c + bp + \overline{a+2}\,p^2\right) = 0$$

$d^2y + \alpha\,dy^2 = p\,dx^2$

$d^3y + 2\alpha\,dy\,d^2y = dp\,dx^2$

$dy\,d^3y + 2\alpha\,dy^2d^2y = dp\,dy\,dx^2$

$a\,d^2y^2 + 2a\alpha\,dy^2d^2y + \alpha^2dy^4 = ap^2dx^4$

$+\beta\,dy^2d^2y + \alpha\beta\,dy^4 = \beta p\,dy^2dx^2$

$2\alpha.\overline{a+1} + \beta = b$, $\alpha^2 + \alpha\beta = c$; $dy = q\,dx$

$dp\,dy + ap^2dx^2 + \beta p\,dy^2 = 0$ $\quad dq + \alpha q^2dx = p\,dx$

$dy =$ $\quad dx = \frac{dq}{p - \alpha q^2}$

$q\,dp + ap^2dx + \beta q^2p\,dx = 0$

$q\,dp + (ap + \beta q^2)\,p\,dx = 0$

$q\,dp + \frac{ap + \beta q^2}{p - \alpha q^2}\,p\,dq = 0$

$p\,dp - \alpha q^2dp + ap^2\frac{dq}{q} + \beta pq\,dq = 0$

$d^2y + \alpha dy^2 = \beta dy^n dx \qquad ; \qquad n\beta dy^{n} dx^{..}$

$dy d^3y + 2\alpha dy^2 d^2y = d\beta dy^{n+1} dx^{2-n} + n\beta^2 dy^{2n} dx^{4-n} - n\alpha\beta dy^{n+2} dx$

$+ a d^2y^2 + 2a\alpha dy^2 d^2y + a\alpha^2 dy^4 = \qquad + a\beta^2 \qquad +$

$+ \quad \beta dy^2 d^2y + \beta dy^4 \qquad + \beta$

$d\beta + \overline{n+a}\,\beta^2 dy^{n-1} dx^{2-n} + \overline{\beta - n\alpha}\, dy = 0$

$dx = u\,dy \;, \; 0 = du\,dy + u\,d^2y \qquad \frac{2-n}{1+n}$

$u d^2y = \beta u dy^n dx^{2-n} - \alpha u dy^2$

$= \beta u^{3-n} dy^2 - \alpha u dy^2 \qquad dy = -\frac{du}{\beta u^{3-n} - \alpha u}$

$du + dy(\beta u^{3-n} - \alpha u) = 0$

$d\beta + \overline{n+\alpha}\,\beta^2 u^{2-n} dy + \overline{\beta - n\alpha}\, dy = 0$

$d\beta = du\,\frac{\overline{n+a}\,\beta^2 u^{2-n} + \overline{\beta - n\alpha}}{\beta u^{3-n} - \alpha u} \quad ; \quad a - n = r \;;\; \beta - n\alpha = \gamma$

$\beta u^{r+1} d\beta - \alpha u\, d\beta = \overline{n+a}\,\beta^2 u^r du + \gamma\, du$

$\alpha = \infty \;;\; a = \delta\alpha \;;\; \beta = \varepsilon\alpha \;,\; \gamma = \overline{\varepsilon - n}\,\alpha$

$-u\,d\beta = \delta\beta^2 u^r du + \overline{\varepsilon - n}\, du \;;$

$d\beta + \delta\beta^2 u^{r-1} du + \overline{\varepsilon - n}\,\frac{du}{u} = 0$

$\text{Log}\,\overline{m-b} = 0.2669383$ $\quad\text{log.}\,\overline{1-b} = \bar{1}.9859629$
subtr. log n $\quad 0.1828967$ - - - add. $\quad 0.1828967$
$\text{Log A} = 0.0840416$ $\quad \text{Log B} = 0.1688596$
$\text{A} = 1{,}213505$ $\quad \text{B} = 1{,}475230$
$\text{A}+\text{B} = 2{,}688735,\ 10+\text{L}.\,\overline{\text{A}+\text{B}} = 10.4295480$
subtr. $\text{Log}\,\overline{m+1-2b}$ $\quad 0.4498178$
$\text{Log Cos Z} = 9.9797302, \text{Z} = 17^\circ{,}3695.$
$\text{Log}\,\overline{m+b} = 0.2816278$ $\quad \text{Log}\,\overline{1+b} = 0.0135977$
subt. Log n $\quad 0.1828967$ - - add. $\quad 0.1828967$
$\text{Log}\,\mathfrak{A} = 0.0987311$ $\quad \text{Log}\,\mathfrak{B} = 0.1964944$
$\mathfrak{A} = 1{,}255253$ $\quad \mathfrak{B} = 1{,}572152.$
$\mathfrak{A}+\mathfrak{B} = 2{,}827405,\ 10+\text{L}.\,\overline{\mathfrak{A}+\mathfrak{B}} = 10.4513880$
subtr. $\text{Log}\,\overline{m+1+2b}$ $\quad 0.4689999$
$\text{Log Cos}\,\mathfrak{Z} = 9.9823881, \mathfrak{Z} = 16^\circ{,}2073.$
Ergo $\text{Z}-\mathfrak{Z} = 1^\circ{,}1622$ cujus $\text{Log} = 0.0652809$
add. $\text{Log}\,\frac{\text{T}}{m-1}$ $\quad 2.8920604$

2.9573413
subtr. Log 360° $\quad 2.5563025$

0.4010388, ergo

$\frac{\overline{\text{Z}-\mathfrak{Z}}.\,\text{T}}{\overline{m-1}.\,360^\circ}$ quam proxime $= 2^d{,}5179 = 2^d{,}12^h{,}25'$ &c.

§. XXVIII.

Sit porro (§. XV. n. 6) $\text{U} = 60^\circ$ adeoque
$\text{Log Sin U} = 9.9375306$
subtr. Log n $\quad 0.1828967$
$\text{Log Sin Y} = 9.7546339$

$$\begin{array}{llll}
Y = 34^\circ{,}6369 & \log \mathrm{Sin}\, U = & 9.9375306 & \\
Z = 85{,}\ 3631 & \log \mathrm{Cos}\, U & 9.6989700 & \text{add.} \\
10 + \log \mathrm{Sin}\, Z = 19.9985762 & \log \overline{m-1} & \bar{1}.9448827 & \\
\text{add. } \log n \quad 0.1828967 & & 19.5813833 & \\
20.1814729 & - \ - \ \text{subtr.} & 20.1814729 & \\
 & \mathrm{Log}\, \gamma = & \bar{1}.3999104 & \&
\end{array}$$

$$\begin{array}{rl}
\gamma = 0{,}251137,\ 1+\gamma = 1{,}251137,\ \log \overline{1+\gamma} = & 0.0973048 \\
\text{add. (§. XXVI.) } \log c & 3.2756619 \\
\mathrm{Log.}\ v = & 3.3729667 \\
v = & 39',20'',2973
\end{array}$$

§. XXIX.

Pro VENERE $m = 1{,}626126$, $\log. m = 0.2111542$, $\log n = 0.1407695$, quamobrem $n = 1{,}382832$, $n+1 = 2{,}382832$, $n-1 = 0{,}382832$, $m-1 = 0{,}626126$, $m+1 = 2{,}626126$, $m+n = 3{,}008958$, $m-n = 0{,}243294$. Hinc pro *Stationibus* (§. XV. n. 1.)

$$\begin{array}{ll}
20 + \log \overline{n+1} = 20.3770934 & \text{cujus dimid.} = 10.1885467 \\
\text{add. } \log n \quad 0.1407695 & - \ - \ \text{subtr.} \quad 0.1407695 \\
20.5178629 & \mathrm{Log\ Cot}\, U = 10.0477772 \\
\text{dim.} = \log \mathrm{Cot} Y = 10.2589315 & U = 138^\circ{,}1452, \\
Y = 28^\circ{,}\ 8501 & \&\ \ Z = 13^\circ{,}0047
\end{array}$$

Hoc facto est (§. XV. n. 3.)

$$\begin{array}{rl}
2Z = 26^\circ{,}0094 \text{ cujus } \mathrm{Log} = & 1.4151303 \\
\text{subtr. } \log \overline{m-1} & \bar{1}.7966617 \\
\mathrm{Log}\ 41^\circ{,}5404 = & 1.6184686.
\end{array}$$

Log.

$$\cancel{y = \frac{d^2z + \alpha dz}{dz^n dx^{2-n}}} \qquad y = \frac{d^2z + \alpha dz}{dz\,dx}$$

$$\cancel{dy = d^3z - \alpha} \qquad dy = \frac{d^3z + 2\alpha dz d^2z}{dz\,dx} - \frac{d^2z.(d^2z + \alpha dz^2)}{dz^2 dx}$$

$$dy = \frac{dz d^3z - d^2z^2 + \alpha dz^2 d^2z}{dz^2 dx}$$

$$y^2dx = + \frac{d^2z^2 + 2\alpha dz^2 d^2z + \alpha^2 dz^4}{dz^2 dx}$$

$$dz d^3z + 3\alpha dz^2 d^2z + \alpha^2 dz^4 - bx^n dz^2 dx^2 = 0$$

$$d^3z + 3\alpha dz d^2z + \alpha^2 dz^3 = bx^n dz dx^2$$

$$d^2z = p dz^n dx^{2-n}$$

$$dp dz^n dx^{2-n} + np^2 dz^{2n-1} dx^{4-2n} + 3\alpha p dz^{n+1} dx^{2-n}$$

$$y = \frac{dz + \alpha dx}{z\,dx}$$

$$dy = \frac{d^2z}{z dx} - \frac{dz^2}{z^2 dx} - \frac{\alpha dz}{z^2}$$

$$y^2 dx = + \frac{dz^2}{z^2 dx} + \frac{2\alpha dz}{z^2} + \frac{\alpha^2 dx}{z^2}$$

$$z^? d^2z + \alpha dz dx + \alpha^2 dx^2 + bx^n z^2 dx^2 = 0$$

$$\frac{d^2y}{y} + \cancel{dy} + X dx^2 = 0 \;;\; y = z N^{\alpha\int X' dx}$$

$$\frac{dy}{y} = \frac{dz}{z} + \alpha X' dx$$

$$\frac{dy^2}{y^2} = \frac{dz^2}{z^2} + 2\alpha\frac{X' dx dz}{z} + \alpha^2 X'^2 dx^2$$

$$\cancel{\frac{d^2y}{y}} \quad \frac{d^2z}{z} + 2\alpha\frac{X' dx dz}{z} + \alpha^2 X'^2 dx^2 + \alpha dX' dx + X dx^2 = 0$$

$$\frac{dz}{z} = du + \int X'' dx, \quad \cancel{\frac{d^2z}{z}} = d^2u + du^2 + 2X'' dx du + X''^2 dx^2 + 2\alpha X' dx du + \cancel{dX'' dx} + \cancel{2\alpha X' X'' dx^2} + \cancel{\alpha^2 X'^2 dx^2} + \cancel{\alpha dX' dx} + X dx^2 = 0$$

$$X'' + \alpha X' = 0 \qquad d^2u + du^2 + X dx^2 = 0$$

$$dX'' = -\alpha dX' \qquad \frac{d^2y}{y} + X dx^2 = 0$$

$$X''^2 = \alpha^2 X'^2$$

$$2\alpha X'' X' = -2\alpha^2 X'^2$$

$$dy = \frac{dz}{z}\,,\quad d^2y = \frac{d^2z}{z} - \frac{dz^2}{z^2}\,;\quad d^3y = \frac{d^3z}{z} - \frac{3dz\,d^2z}{z^2} + \frac{dz^3}{z^3}$$

$$d^4y = \frac{d^4z}{z} - \frac{4dz\,d^3z}{z^2} - \frac{3d^2z^2}{z^2} + \frac{6dz^2d^2z}{z^3} - \frac{dz^4}{z^4}$$

$$\frac{a\,dy}{dx} = \frac{a\,dz}{z\,dx}$$

$$+\frac{b\,d^2y}{dx^2} = -\frac{b\,dz^2}{z^2dx^2} + \frac{b\,d^2z}{z\,dx^2}$$

$$+\frac{c\,d^3y}{dx^3} = +\frac{c\,dz^3}{z^3dx^3} - \frac{3c\,dz\,d^2z}{z^2dx^3} + \frac{c\,d^3z}{z\,dx^3}$$

$$+\frac{e\,d^4y}{dx^4} = -\frac{e\,dz^4}{z^4dx^4} + \text{\&c}$$

$$dz + z^2dx + X\,dx = 0 \qquad z = \frac{dy}{y\,dx} + X'$$

$$dz = \frac{d^2y}{y\,dx} - \frac{dy^2}{y^2dx} + dX'$$

$$\frac{d^2y}{y\,dx} - \frac{dy^2}{y^2dx} + dX' + X\,dx$$
$$+\frac{dy^2}{y^2dx} + \frac{2X'dy}{y} + X'^2dx$$

$$\frac{d^2y}{dx} + 2X'dy + y\left(dX' + X'^2dx + X\,dx\right) = 0$$

Eul. §863. $2X' = \dfrac{N + \frac{dM}{2dx}}{M}\,;\ C = 0\,;\ \dfrac{dN - \frac{NdM}{2M}}{M} = dX + X\ldots$

$$\frac{MdN - NdM}{M^2} = d.\frac{N}{M} = d.\left(2X' - \frac{dM}{2Mdx}\right) = 2dX' - \frac{d^2M}{2Mdx} + \frac{dM^2}{2M^2dx}$$

$$d.\frac{N}{M} + \frac{1}{2}\frac{NdM}{M^2} = dX' + X'^2dx + X\,dx$$
$$= 2dX' - \frac{d^2M}{2Mdx} + \frac{dM^2}{4M^2dx} + \frac{X'dM}{M}$$

$$\frac{dM}{2M} = du,\quad \frac{d^2M}{2M} = d^2u + du^2$$

$$\cancel{\frac{dM}{2M}.\left(2X' - \frac{dM}{2Mdx}\right)} \qquad -\frac{d^2M}{2M} + \frac{dM^2}{2M^2} = -d^2u - du^2$$

$$-\frac{1}{4}\frac{dM^2}{M^2} \qquad X' = \frac{2du}{dx}$$

$$dX' + 2X'du = \frac{d^2u}{dx} + \frac{du^2}{dx} + X'^2dx + X\,dx \qquad dX' = \frac{2d^2u}{dx}$$

$$X' = \frac{dM}{Mdx}\,;\ dX' = \frac{d^2M}{Mdx} - \frac{dM^2}{M^2dx} \qquad -\frac{d^2u}{dx} + \frac{du^2}{dx} + X\,dx = 0$$

$$-du = \frac{dv}{v}$$

$$\cancel{\frac{3}{2}\frac{dM}{Mdx} - \frac{?\,dM^2}{M^2dx}} = \frac{d^2M}{2Mdx} - \frac{3}{4}\frac{dM^2}{M^2dx} - X\,dx = 0 \qquad -\frac{d^2u}{dx} = \frac{d^2v}{v\,dx} - \frac{dv^2}{v^2}$$

$$\frac{d^2v}{v\,dx} + X\,dx =$$

$$\begin{array}{llll} \text{Log } 360^\circ = 2.5563025 & & 2Y = & 57^\circ,7002 \\ \text{fubt.log}\, \overline{m-1} & \overline{1}.7966617 & \text{fubt.}\ \frac{2Z}{m-1} & 41,5404 \\ & 2.7596408 & & 16^\circ,1598, \textit{Arc. Retr.} \\ \frac{360^\circ}{m-1} = 574^\circ,9642 & - - & \text{add.} & 574,9642 \\ & & & 591^\circ,1240; \end{array}$$

Arcus motu *directo* a Venere percurfus.

Cum Tellus refpectu Veneris fit Planeta a Sole remotior fumi debet $T = 365,2565$, ideoque log. $T = 2.5625979$

$$\text{add. Log}\ \frac{2Z}{m-1} \quad 1.6184686$$
$$4.1810665$$

& demto Log $360^\circ = 2.5563025$, fit Log $\frac{Z.T}{m-1.\ 180^\circ} =$ $= 1.6247640$ cui refpondent dies 42, 1467 *Durationem retrogradationis* definientes. Jam vero Log $\frac{T}{m-1} = 2.7659362$ & exinde *Periodus Phænomenorum* $= 583^d,3594$, unde fubtrahatur Tempus retrogradationis & obtinebitur *Duratio Progreffionis* $= 541^d,2127$. Hinc quoque Tempus inter conjunctionem fuperiorem & inferiorem intercedens eft dierum 291,6797, defcriptis interim arcubus a Tellure $287^\circ,4821$ & a Venere $467^\circ,4821$. Ergo ab una conjunctione fuperiori (inferiori) ad fubfequentem percurrit Tellus arcum $574^\circ,9642$ & Venus arcum $934^\circ,9642$.

§. XXX.

Ut obtineatur celeritas ♀:is adparens in Syzygiis (§. XV. n. 5.), ubi c denotat celeritatem heliocentricam Telluris, eodem modo ac antea, fiat hic calculus.

Log 1296000 = 6.1126050
subtr. Log T $\underline{2.5625979}$
Log c = 3.5500071
add. Log $\overline{m+n}$ $\underline{0.4784160}$
4.0284231
subtr. Log $\overline{n+1}$ $\underline{0.3770934}$
Log v = 3.6513297

Log $\overline{m-n}$ = $\bar{1}$.3861314
- - - add. $\underline{3.5500071}$
2.9361385
subtr. Log $\overline{n-1}$ $\underline{\bar{1}.5830082}$
Log $-v$ = 3.3531303

max. veloc. directa vel $v = 1^\circ,14',40'',5335$ & Max. vel. retr. vel $-v = 37',34'',9159$.

§. XXXI.

Quærere quoque lubet, quamdiu motus adparens Veneris (spatio 24^h) non sit major $1'$ (§. XV. 7.). Quem in finem exprimatur (cfr. §. XIX.) c minutis primis, ita ut

Log c = 1.7718558, & Log $\frac{v}{c}$ = Log b = $-$ Log c = $\bar{2}$.2281442. Unde $b = 0,016910$, $m-b = 1,609216$ ($= 256 \times 0,006286$), $m+b = 1,643036$, $1+b = 1,01691$, $1-b = 0,98309$, $m+1+2b = 2,659946$, $m+1-2b = 2,592306$. Ergo (§. XV. 8.).

Log $\overline{m-b}$ = 0.2066144
subtr. Log. n $\underline{0.1407695}$
Log A = 0.0658449
A = 1,163710

Log $\overline{1-b}$ = $\bar{1}$.9925933
- - - add. $\underline{0.1407695}$
Log B = 0.1333628
B = 1,359449.

A+B = 2,523159, 10+L. $\overline{A+B}$ = 10.4019445
subtr. Log $\overline{m+1-2b}$ $\underline{0.4136862}$
Log Cos Z = 9.9882583, Z = 13°,2633.

Log $\overline{m+b}$ = 0.2156471
subt. Log n $\underline{0.1407695}$ - -
Log $\mathfrak{A}$ = 0.0748776
$\mathfrak{A}$ = 1,188167

Log $\overline{1+b}$ = 0.0072825
add. $\underline{0.1407695}$
Log $\mathfrak{B}$ = 0.1480520
$\mathfrak{B}$ = 1,406216

$\mathfrak{A}+\mathfrak{B}$

Secundum obs. circa transitum ♀ is p ☉ 1761 factas motus horarius apparens ♀ in conjunctione est = 4'.0".92. = 4'.1". adeoque ~~ess~~ motus diurnus f. 24^h, maximam ~~w~~ exprimens erit 96'24" ~~= 1°36'24~~. Tantus vero ~~ab~~ observationibus dissensus ~~discrepantia~~ inutilem prorsus arguere videtur calculum, qui sub simpliciori hac hyp. ~~ab~~ institui.

$$N^{\int 2Qdx}\left(C - \int N^{\dots} dx\right) + \frac{1}{y - Q} = 0$$

$$y - Q + \frac{N^{-2\int Q dx}}{C - \int N^{-2\int Q dx} dx} = 0 \qquad y + N^{-\int y dx}\left(A + \int N^{\int y dx} . X dx\right)$$

$$dy + y^2 dx + X dx = 0 \qquad A = 0 = C;$$

$$y + \varphi\psi = 0 \quad \varphi = -\frac{y}{\psi} \qquad Q + 1 + N^{-\int y dx} \int N^{2\int y dx} X dx = 0$$

$$y dx = -\frac{d\psi}{\psi}$$

$$\psi = N^{-\int y dx}$$

$$\psi d\varphi = X dx$$

$$\varphi = \int N^{\int y dx} X dx$$

$$n = -2, \qquad z = M; \quad Z dz = -\frac{d^3 M}{2 M dx^2}$$

$$M = y^2$$

$$N = -\frac{dM}{2dx} = -\frac{y dy}{dx} =$$

$$-\frac{y^2 dy}{dx} - \frac{y^2 dy}{dx}$$

$$-\frac{y^2 dy}{2dx^2} + \frac{1}{2} y^2 \left(C N^{-3\int \frac{dy}{y}} + \cancel{B} \frac{dy^2}{dx^2}\right) = A$$

$$\frac{d^3 M}{2dx^2} + 2Q dM + M dQ = 0$$

$$dQ = -2Q\frac{dM}{M} - \frac{d^3 M}{2 M dx^2}; \quad Q = M^{-2}\left(C - \int \frac{M d^3 M}{2 dx^2}\right)$$

$$Q = \frac{C}{M^2} - \frac{d^2 M}{2 M dx^2} + \frac{dM^2}{4 M^2 dx^2}$$

$$M d^2 M$$

$$M d^3 M + dM d^2 M$$

$$\int M d^3 M = M d^2 M - \int dM d^2 M \qquad \frac{dM}{M} = \alpha \frac{dv}{v}$$

$$= M d^2 M - \tfrac{1}{2} dM^2 \qquad \frac{d^2 M}{2M} = \alpha \frac{d^2 v}{2v} + \alpha^2 \frac{dv^2}{2v^2} - \alpha \frac{dv^2}{2v^2}$$

$$\cancel{-\frac{d^2 M}{2 M dx^2}} = -\frac{\alpha}{2}\cancel{\frac{d^2 v}{dx^2}} - \frac{\alpha^2}{2}\cancel{\frac{dv^2}{dx^2}} \qquad \frac{dM^2}{4 M^2 dx^2} = -\frac{\alpha^2 dv^2}{4 v^2}$$

$$\cancel{+\frac{dM^2}{4 M^2 dx^2}} = \qquad + \frac{\alpha^2}{4}\cancel{\frac{dv^2}{dx^2}}$$

$$\frac{\alpha^2}{4} - \frac{\alpha}{2} = 0$$

$$\frac{\alpha}{2} = 1$$

$$\alpha = 2$$

$\mathfrak{A}+\mathfrak{B}=2{,}594383$, $10+\mathrm{L}.\overline{\mathfrak{A}+\mathfrak{B}}=10.4140341$
subtr. Log. $\overline{m+1+2b}$ $\quad 0.4248728$

Log Cof Z $=9.9891613$, $\mathfrak{Z}=12^\circ{,}7475$.

ergo $Z-\mathfrak{Z}=0{,}5158$ cujus log $=\bar{1}.7124813$

add. Log $\frac{T}{m-1}$ $\quad 2.7659362$

2.4784175

subtr log 360° $\quad 2.5563025$

$\bar{1}.9221150$, cui respondent $0^d{,}8358 = 20^h{,}3'{,}33''$ &c. $=$ *Durationi Stationis* quaesitae.

§. XXXII.

Quomodo inveniri possit intervallum temporis, inter momentum *Digressionis maximae* Planetae inferioris & momentum *Stationis* proxime praecedentis l. subsequentis, diximus §. XV. n. 9. Coll. 1. Facta igitur adplicatione ad Venerem, est $\mathrm{Log\,Sin\,Y} = 10 + \mathrm{Log}\frac{1}{n} = 9.8592305$ & huic respondens arcus $46^\circ, 18'{,}9271$ *maxima Veneris elongatio*,(*) cujus complementum $=43^\circ{,}6845$ minutum angulo pro Stationibus $Z = 13^\circ{,}0047$ facit $\zeta = 30^\circ{,}6798$ cujus

Log $= 1.4868525$

add. Log. $\frac{T}{m-1}$ $\quad 2.7659362$

4.2527887

subtr. Log 360° $\quad 2.5563025$

1.6964862 cui respondent $49^d{,}7149$

$49^d,7149$ = intervallo temporis, quo maxima Veneris elongatio antecedit vel sequitur momentum Stationis.

(*). Apud D:um DE LA CAILLE (l. c. Sect. I. Part. I. Art. XIII. Tab.) est minima dist. ♁:is a ☉: maximam distantiam ♀:is a ☉ :: 9998:7233 adeoque $n = \frac{9998}{7233}$ & maxima digressio ♀:is $46°,20',3137$.

§. XXXIII.

Ponatur, compendii caussa, datus esse $Y = 18°$, qui, respectu Veneris & Mercurii, est angulus ad Tellurem & quæratur quanta sit *velocitas Veneris adparens* in illo loco (§. XV. 6.). Jam igitur Log Sin $Y = 9.4899824$ addatur Log $n = 0.1407695$ adeo ut (ob Sin $U = n$. Sin Y) Log Sin $U = 9.6307519$ & quando Venus est directa $U = 25°,17',8512$, consequenter $Z = 136°,42',1488$.

		Log. Sin U =	9.6307519	add.
		Log. Cos U	9.9562170	
10 + Log Sin Z =	19.8361891	Log $\overline{m-1}$	$\bar{1}.7966617$	
add. Log n	0.1407695		19.3836306	
	19.9769586	- - subtr.	19.9769586	
		Log γ =	$\bar{1}.4066720$,	&

$\gamma = 0,255077$, $1+\gamma = 1,255077$, Log $\overline{1+\gamma} = 0.0986703$

add. (§. XXX.) Log c 3.5500071

Log $v = 3.6486774$.

$v = 4453'',2533$.

Ergo velocitas ♀:is adparens (ubi $Y = 18°$ & motus est directus) æqualis $1°,14',13''$ quam proxime.

§. XXXIV.

Pro MERCURIO sunt $m = 4,152095$, Log $m = 0.6182672$, log $n = 0.4121781$, $n = 2,583319$, $n+1 =$

3,583319,

$dy + (y^2 + X)dx = 0 \quad u^{2}v$

$u\frac{dv}{dx} = 2vuy + (y^2+X)v\frac{du}{dy}$

$2\cancel{v}uX + \cancel{v}y\frac{du}{dy} = 0, \quad u\frac{dv}{dx} = vX\frac{du}{dy}; \quad \frac{dv}{vXdx} = \frac{du}{udy}$

$\frac{dv}{udy} = -\frac{\cancel{v}}{X} \qquad 2\cancel{v}uy + X\cancel{v}\frac{du}{dy} = 0$

$X = -2u\frac{ydy}{du}$

$du \qquad udv + vdu = 2vuydx$

$\frac{dY}{y} - 2X' \quad v = 2y \quad y = -\frac{2dy}{y} + \frac{dX'}{X} \quad y = \frac{2Xdx}{X'}$

$Luv = \int 2ydx$

$u+v; \quad \frac{dv}{dx} = \overline{y^2+X}\frac{du}{dy} + 2y.\overline{u+v}$

$y^2\frac{du}{dy} + 2y^2u = 0; \quad y\frac{du}{dy} + 2u = 0,$

$\frac{du}{dy} = -\frac{2u}{y}$

$\frac{dv}{dx} = 2yv - \frac{2u}{y}X \qquad v = aX \quad u = by^2 + cy$

$\frac{2u}{y} = 2by + 2c$

$\frac{adX}{Xdx} = 2a - 2by - 2c \qquad ydx = -\frac{dy}{y} - \frac{y}{X}dx$

$u = \alpha y^2 + \beta y; \; (2v - 2\alpha X)y - 2\beta X = \frac{dv}{dx}$

$PN^{\alpha y} - N^{\alpha y}\frac{dP}{dx} = N^{\alpha y}(\alpha y^2 + XP + 2yP)$

$dv = pdx + qdy; \quad N^{v}(dy + \overline{y^2+X}dx) = 0$

$p = 2y + \overline{y^2+X}q; \quad pdx = 2ydx - qdy \qquad v = C - Ly - \int\frac{Xdx}{y}$

$d\!v = 2dydx \qquad dv = 2ydx = -\frac{2dy}{y} - \frac{2Xdx}{y}$

$d^2v + 2dx^2(y^2+X) = 0 \qquad y = \frac{dv}{2dx} \qquad v(dy + \overline{y^2+X}dx) = 0$

$d^2v + \frac{dv^2}{2} + 2Xdx^2 = 0 \qquad p = 2yv + \overline{y^2+X}q \qquad v = \frac{Y - 2\int Xdx}{y}$

$pdx = 2yv_x - qdy$

$q = -\frac{2X}{y} \qquad \frac{dv}{v} = 2ydx \qquad v = \frac{Y - 2\int Xdx}{y}$

$-\frac{2dy}{y} = \frac{ydY - 2(Y - 2\int Xdx)dy}{y^2}$

$dy + (ay^2 + by + X\cancel{dx})\,dx = 0$

$N^v;\ dv = p\,dx + q\,dy$

$p = (ay^2 + by + X)q + 2ay + b$

$\cancel{p\,dx + q\,dy = dv}\quad dv = \overline{2ay + b}\,dx;$

$\cancel{p\ d\ i}\qquad y\,dv = \overline{2ay^2 + by}\,dx$

$-2dy - 2X\,dx = \overline{2ay^2 + 2by}\,dx$

$y\,dv + 2dy + 2X\,dx + by\,dx = 0$

$2ay\,dv + 4a\,dy + 4aX\,dx + 2aby\,dx = 0$

$b\,dv - 2aby\,dx - b^2\,dx = 0$

$(2ay + b)\,dv + 4a\,dy + (4aX + b^2)\,dx = 0$

$\cancel{dy + dv}\ dy + \frac{y\,dv}{2} - \frac{b\,dv}{4a} + \frac{4aX + b^2}{4a}\,dx = 0$

$b = 0,\ dy + \frac{y\,dv}{2} + X\,dx = 0$

$\cancel{dy + ay^2\,dx}$

$y = N^{\int P\,dx} \int N^{\int Q\,dx}\,dx$

$dy = N^{\int P\,dx} P\,dx \int N^{\int Q\,dx} + N^{\int P\,dx + \int Q\,dx}\,dx$

$y^2\,dx = N^{2\int P\,dx} \left(\int N^{\int Q\,dx}\,dx\right)^2$

$= N^{\int P\,dx}\ z\,dx$

$+ P\int N^{\int Q\,dx}\,dx + N^{\int Q\,dx} + N^{\int P\,dx}\left(\int N^{\int Q\,dx}\,dx\right)^2$

$y = 1\int \cancel{X}\,dx + z \qquad dz + z^2\,dx + 2z\,dx\int X\,dx + dx\left(\int X\,dx\right)^2$

3,583319, $n-1=1,583319$, $m+1=5,152095$ ($=585 \times 0,008807$), $m-1=3,152095$, $m+n=6,735414$, $m-n=1,568776$. Ergo (§. XV. 1.)

$$
\begin{array}{llll}
20+\log \overline{n+1}=20.5542855 & \text{- - dimid.} & = 10.2771427 \\
\text{add. Log } n \quad 0.4121781 & \text{- - - subtr.} & 0.4121781 \\
\hline
\quad 20.9664636 & \text{Log Cot. U} = & 9.8649646 \\
\text{dim.}=\text{L. Cot. Y}=10.4832318 & & U=126^\circ,2326. \\
\quad Y=18^\circ,1945, & \text{proinde} & Z=35^\circ,5729.
\end{array}
$$

Ne videar prorsus frustra notasse (§. cit.) formulas Sin Z & Cos Z, proferam hic earum calculum; scil.

$$
\begin{array}{llll}
\frac{1}{2}\log \overline{n+1} = & 0.2771427 & \log n = & 0.4121781 \\
\text{add.} 10+\text{l.}\overline{n-1} & 10.1995684 & \text{add.} 10+\text{l.}\overline{m+n} & 10.8283643 \\
\hline
 & 10.4767111 & & 11.2405424 \\
\text{subtr. log } \overline{m+1} & 0.7119839 & \text{subt. l.} m+\text{l.}\overline{m+1} & 1.3302511 \\
\hline
\text{Log Sin Z} = & 9.7647272 & \text{Log Cos Z} = & 9.9102913 \\
Z = & 35^\circ,5729 & Z = & 35^\circ,5729.
\end{array}
$$

$$
\begin{array}{ll}
\text{Porro } 2Z = 71^\circ,1458, \text{ cujus Log} = & 1.8521493 \\
\text{subtr. Log } \overline{m-1} & 0.4985993 \\
\hline
\text{Log. } 22^\circ,5710 = & 1.3535500
\end{array}
$$

$$
\begin{array}{llll}
\text{Log.} 360^\circ = & 2.5563025 & 2Y = & 36^\circ,3890 \\
\text{subt. L.} \overline{m-1} & 0.4985993 & \text{subt.} \frac{2Z}{m-1} & 22,5710 \\
\hline
 & 2.0577032 & & 13^\circ,8180 = \textit{Arc. Retr.} \\
\frac{360^\circ}{m-1} = & 114^\circ,2098 & \text{- - add.} & 114,2098 \\
\hline
 & & & 128,0278 = \textit{Arc. Dir.}
\end{array}
$$

Dein-

Deinde (§. cit. 2. cfr. XVI.)

$$\text{Log } T = 2.5625979$$

$$\text{add. Log.}\frac{2Z}{m-1} \quad 1.3535500$$

$$3.9161479$$

$$\text{subtr. log } 360^\circ \quad 2.5563025$$

$$\text{Log.}\frac{Z.T}{m-1.180^\circ} = 1.3598454 \qquad \text{Log}\frac{T}{m-1} = 2.0639986$$

$$\text{Per. Phæn.} = 115^d,8774$$

Duratio retrogr. $= 22^d,9005$ - - - subtr. $22,\ 9005$

$$\text{Duratio progress.} = 92^d,9769.$$

Facile hinc patet, Mercurium a conjunctione superiori ad inferiorem & vicissim pervenire intervallo temporis $57^d,9387$, quo arcus a Tellure percursus est $57^\circ,1049$ & a ☿:io $237^\circ,1048$, adeoque Periodo Phæn. emeritur Mercurius arcum $474^\circ,2096$, Tellus arcum $114^\circ,2098$, qui supra jam inventus exstat.

§. XXXV.

Adparentes Mercurii celeritates in Syzygiis sequenti calculo innotescunt.

$$\text{Log}\,\overline{m+n} = 0.8283643 \qquad \text{Log.}\,\overline{m-n} = 0.1955610$$

$$\text{add.(§.XXX.) log}\,c \quad 3.5500071 \quad - - - \text{add.} \quad 3.5500071$$

$$4.3783714 \qquad 3.7455681$$

$$\text{subtr. log.}\,\overline{n+1} \quad 0.5542855 \quad \text{subtr. log}\,\overline{n-1} \quad 0.1995684$$

$$\text{Log } v = 3.8240859 \qquad \text{Log } -v = 3.5459997$$

$$v = 1^\circ,51',9'',3871 \qquad -v = 58',35'',6019.$$

§. XXXVI.

Placet denique pro Mercurio inquirere durationem *Stationis* eo sensu, cujus supra mentionem fecimus. Sunt igitur $c, v, b, 1+b, 1-b$ ut §. XXXI, & $m-b = 4,135185$,

$m+b$

$dy + aydx + bydx + cdx = 0$

$\cancel{y = z \int Xdx}$

$\cancel{dz + az^2dx = cdx}$

$Pdz + zdP + dQ$

$+ az^2P^2dx + 2a$

$Pdz + zdP + dQ$

$+P^2z^2dx + \cancel{2zPQ}dx + Q^2dx$

$+ Xdx$

$dP + 2PQdx = 0$

$dQ + Xdx = 0$

$dz + z^2 Pdx + \frac{Q^2}{P}dx = 0$

$Q = -\frac{dP}{2Pdx}$, $du = Pdx$, $\frac{d^2u}{dx^2} = \frac{dP}{Pdx} = -2Q$

$dz + z^2du + \frac{d^2u^2}{4dx^3} = 0$; $\frac{d^2u}{} = 4adx^2$

$dy + y^2dx + \frac{d^3u}{2dx^2} = 0$

$d^3u = 0$

$d^2u = Adx^2$

$dz + z^2du + \frac{dx}{4}\left(d.\frac{du}{dx}\right)^2 = 0$

$du = Bdx + Axdx$

$u = C + Bx + \frac{1}{2}Ax^2$

$\frac{d^3u}{dx^2} = adx$

$\frac{d^2u}{dx} = Adx + axdx$

$dy + y^2dx + \frac{d^3u}{2dudx} - \frac{d^2u^2}{2du^2dx} = 0$

$dz + z^2du + \frac{d^2u^2}{4dudx} : du = 0$

$dz + z^2du + \frac{d^2u^2}{4du^3} = 0$

$y = zP + Q$

$dy = Pdz + zdP + dQ$

$dy + y^2dx + Xdx = 0$

$y = zP + Q$

$Q = A - \int Xdx$

$\frac{dP}{P} = -2Qdx$

$lP = lC + 2\int dx \int Xdx - 2Ax$

$P = CN^{2\int dx \int Xdx - 2Ax}$

$\frac{du}{dx} = A + 4ax = P$

$X = -\frac{dQ}{dx}$

$Q = -\frac{d^2u}{2dx}$, $-\frac{dQ}{dx} = +\frac{d^3u}{2dx^2}$

$Pdx = du$, $X = -\frac{dQ}{dx}$

$\frac{dP}{2Pdx} = -Q = \frac{d^2u}{2dudx}$

$-\frac{dQ}{dx} = X = \frac{d^3u}{2dudx^2} - \frac{d^2u^2}{2du^2dx^2}$

$y = \frac{P}{z} + Q$

$dQ + Xdx = 0$

$\frac{dP}{z} - \frac{Pdz}{z^2} + dQ$

$2Qdx = \frac{dz}{z}$

$+\frac{2PQdx}{z} + \frac{P^2dx}{z^2} + Q^2dx$

$+ Xdx$

$\frac{dx}{z} = \frac{dz}{2z^2Q}$

$dP + \frac{P^2dz}{2z^2Q} + \frac{1}{2}Qdz = 0$; $X = -\frac{dQ}{dx} = -\frac{d^2z}{}$

$Q = \frac{dz}{2z\,dx} = -\frac{dxd^2u}{du^2} : \frac{2dx^2}{du}$

$\frac{dz}{2z^2Q} = du = \frac{dx}{z}$

$-Q = +\frac{d^2u}{2dxdu}$ $\frac{dz}{2z^2} : \frac{dz}{2zdx} = \frac{dx}{z}$

$z = \frac{dx}{du}$

$dz = -\frac{dxd^2u}{du^2}$ $-\frac{dQ}{dx} = +\frac{d^3u}{2dx^2du} - \frac{d^2u^2}{}$

$\frac{1}{2}Qdz = \frac{dz^2}{4zdx} = \frac{dx^2d^2u^2}{du^4} : \frac{4dx^2}{du} = \frac{d^2u^2}{4du^3}$

$d^2u^2 = 4a^2du^4$

$dP + P^2du + \frac{d^2u^2}{4du^3} = 0$

$d^2u = 2adu^2$

$dy + y^2dx + \frac{d^3u}{2dxdu} - \frac{d^2u^2}{2dxdu^2} = 0$ $C\frac{du}{dx} = N^{2au}$

$x = A + CN^{-2au}$

$dy + y^2dx + \frac{adx}{} = 0$ $d^3u = 4a^2du^3$

$\cancel{y = (z - A)N^{mx}}$

$\cancel{dy = N^{mx}dz + mdx(z - A)N^{mx}}$

$\cancel{+ (z - A)^2N^{2mx}dx} + \frac{adx}{x} = 0$

$dy + y^2dx = 0$; $Q = \alpha$; $\frac{dz}{z} = 2\alpha dx$

$z = \beta N^{2\alpha x}$

$\cancel{du = N^{2a}}$

$\cancel{du = dx}$

$du = \frac{N^{-2\alpha x}dx}{\beta}$; $u = \frac{N^{-2\alpha x}}{\gamma}$

$y = \frac{P}{\beta N^{2\alpha x}} + \alpha$

$Lyu = -2\alpha x$

$\frac{dx}{\beta N^{2\alpha x}} = du = \frac{N^{-2\alpha x}dx}{\beta}$

$dy = \frac{dP}{\beta N^{2\alpha x}} - \cancel{\frac{2\alpha Pdx}{\beta N^{2\alpha x}}}$

$u = -\frac{N^{-2\alpha x}}{2\alpha\beta}$

$+y^2dx = \frac{P^2dx}{\beta^2N^{4\alpha x}} + \cancel{\frac{2\alpha Pdx}{\beta N^{2\alpha x}}} + \alpha^2dx$

$-2\alpha\beta = 1$

$dP + \frac{P^2dx}{\beta N^{2\alpha x}} + \alpha^2\beta N^{2\alpha x}dx = 0$

$du = N^{-2\alpha x}$

$\frac{du}{u} = -2\alpha dx\, N^{-2\alpha x}$

$m+b = 4,169005$, $m+1+2b = 5,185915$, $m+1-2b = 5,118275$. Ergo

$\text{Log}\,\overline{m-b} = 0.6164950$ $\quad \text{log.}\,\overline{1-b} = \bar{1}.9925933$
$\text{subt. log}\,n = 0.4121781$ - - $\text{add.}\ 0.4121781$
$\text{Log A} = 0.2043169$ $\quad \text{Log B} = 0.4047714$
$\text{A} = 1,600726$ $\quad \text{B} = 2,539636$
$\text{A}+\text{B} = 4,140362$, $10+\text{log}\,\overline{\text{A}+\text{B}} = 10.6170383$
$\text{subtr. log}\,\overline{m+1-2b}\ \ 0.7091236$
$\text{Log Cos Z} = 9.9079147$, $\text{Z} = 36^\circ,0078$
$\text{Log}\,\overline{m+b} = 0.6200324$ $\quad \text{Log}\,\overline{1+b}\ \ 0.0072825$
$\text{subt. log}\,n\ \ 0.4121781$ - - $\text{add.}\ 0.4121781$
$\text{Log}\,\mathfrak{A} = 0.2078543$ $\quad \text{Log}\,\mathfrak{B} = 0.4194606$
$\mathfrak{A} = 1,613817$ $\quad \mathfrak{B} = 2,627003$.
$\mathfrak{A}+\mathfrak{B} = 4,24082$, $10+\text{log}\,\overline{\mathfrak{A}+\mathfrak{B}} = 10.6274499$
$\text{subtr. log}\,\overline{m+1+2b}\ \ 0.7148254$
$\text{Log Cos}\,\mathfrak{Z} = 9.9126245$, $\mathfrak{Z} = 35^\circ,1391$.

$\text{Z}-\mathfrak{Z} = 0.8687$ cujus logarithmo $= \bar{1}.9388698$ si addatur $\text{log}\,\frac{\text{T}}{m-1} = 2.0639986$ & ab illorum summa auferatur $\text{log}\ 360^\circ = 2.5563025$, fit $\text{Log}\,\frac{\text{Z}-\mathfrak{Z}}{m-1.\,360^\circ}.\,\text{T} = \bar{1}.4465659$ & huic respondens *Duratio Stationis* $= 0^d,2796 = 6^h, 42', 37''$ &c.

§. XXXVII.

Quando Mercurius maxime a Sole digreditur, est (§. XV. n. 9. cfr. §. VIII. Cor. 5) $\text{Sin Y} = \frac{1}{n}$ & $\text{log Sin Y} = 9.5878219$, cui respondent $22^\circ,46',4463$ (*). Hac autem posita elongatione maxima, inveniuntur, rite subducto calculo, dies

 10,1885,

10,1885, quibus illa Stationis momentum antecedit vel subsequitur.

(*) Quod ad maximam Mercurii elongationem adtinet, illam nonnulli nimis magnam, meo quidem judicio, statuere videntur. Sic D:us KEILL Astr. p. 339 dicit eam adæquare circiter gradus 33, quamvis vix crediderim, illum id studio contendisse, præcipue cum p. 346 aliter pronunciet. Verum quidem est, quod hæ digressiones non semper æquales deprehendantur; nam, ut refert Cel. DE LA LANDE Astr. §. 837, p. 395, PTOLEMÆUS inter 16°,8′ & 28°,37′ atque KEPLER inter 17°,33′ & 28°,31′, maximas Mercurii digressiones esse opinantur; interim tamen Cel. WOLFFIVS (Anf. gr. Astr. §. 305.) & KÆSTNER (Anf. gr. Astr. §. 202.) dicunt elongationem ☿:ii > 28° nunquam esse. Quicquid sit, hunc quoque terminum justo majorem esse arbitror, quia secundum Cel. D:um DE LA CAILLE (l. c. Sect. I. Cap. 2. Art. XIII. Tab.) est $\frac{\text{minima dist. ♁ a ☉}}{\text{max. dist. ☿ a ☉}}$ $= \frac{9998}{3871} = n$ minimæ, adeoque maximarum maxima $Y =$ 22°,46′,7394.

§. XXXVIII.

Si tandem hic quoque ponatur (§. XV. 6. cfr. XXXIII.) $Y = 18°$, erit $\log n + \log \text{Sin } Y = \text{Log Sin } U = 9.9021605$, & sic cum Mercurius est directus, $U = 52°,58',0252$.

Ergo $Z = 109°,1',9748$,		Log sin U =	9.9021605	add.
		Log Cos U	9.7797939	
10 + log Sin Z =	19.9755841	log $m - 1$	0.4985993	
add. log n	0.4121781		20.1805537	
	20.3877622	- - subt.	20.3877622	
		Log γ =	$\bar{1}$.7927915	

Ideo-

$dy\,d^3y + a\,d^2y^2 + b\,dx\,d^2y\,dy + c\,dx^2dy^2 = 0$

$d^2y = p\,dx\,dy\,,\quad d^3y = dp\,dx\,dy + p^2dx^2dy$

$dp\,dx\,dy^2 + dx^2dy^2(\overline{a+1}\,p^2 + bp + c) = 0$

$\frac{dp}{c+bp+\overline{a+1}\,p^2} + dx = 0 \qquad c + \overline{B+cA}\,p + ABp^2$

$\frac{\alpha\,dp}{1+Ap} + \frac{\beta\,dp}{c+Bp} + dx = 0 \qquad \alpha c + \beta = 1 \quad \alpha B + \beta A = 0$

$B + cA = b,\quad AB = a+1$

$B - \tfrac12 b = \pm\tfrac12\sqrt{b^2 - 4c.\overline{a+1}} \qquad B^2 + \overline{a+1}\,c = bB$

$B = \frac{b+n}{2}\ ;\ A = \frac{b-B}{c} = \frac{b-n}{2c}\ ;\ cA - B = -n$

$\alpha cA + \beta A = A \qquad \alpha = \frac{b-n}{2nc}\,,\ \beta = 1 + \frac{b-n}{2n} = \cancel{\frac{n+b}{2n}}$

$\alpha B + \beta A = 0$

$\alpha(cA - B) = A \qquad \beta = \frac{b+n}{2n}$

$\frac{b-n}{2nc}\cdot\frac{dp}{1+\frac{b-n}{2c}p} - \frac{b-n}{2n}\,\frac{dp}{c+\frac{b+n}{2}p} + dx = 0$

$\frac1n L\left(1 + \frac{b-n}{2c}p\right) - \cancel{\frac{b+n}{2n.b+n}}\,L\left(c + \frac{b+n}{2}p\right) + x + \cancel{\frac{1LCc}{n}} = 0$

$\cancel{bn + b^2} \qquad \cancel{\frac{b-n}{b+n} = \frac{\overline{b-n}^2}{b^2-n^2} = \frac{\overline{b-n}^2}{4c.\overline{a+1}}}$

$\cancel{b^2 - 2nb + n^2 = 2b}$

$L\frac{c + \frac{b+n}{2}p}{1 + \frac{b-n}{2c}p} = LC.N^{nx}$

$du\,d^3u - d^2u^2 - 2\alpha\,du^2dx^2 = 0$

$\frac{c + \frac{b+n}{2c}p}{1 + \frac{b-n}{2c}p} = CN^{nx} \qquad a = -1,\ b = 0\ ;\ c = -2\alpha$

$dp = 2\alpha\,dx$

$p = A + 2\alpha x = \frac{d^2u}{dx\,du}$

$L\frac{du}{dx} = Ax + \alpha x^2$

$BN^{Ax+\alpha x^2}dx = du \qquad A^2dx = \frac{A\gamma\,du}{\gamma+\delta u}$

$\alpha = 0\ ;\ \frac{u}{B} + C = \frac1A N^{Ax}$

$\cancel{Adx}\ Ax = L\,\gamma+\delta u$

$dy + y\,dx + \ldots = 0$;

$P\,dz + z\,dP + dQ + X\,dx = 0$

$P^2z^2dx + 2PQz\,dx + Q^2dx$

$\cancel{dz + z^2P\,dx + \frac{Q^2dx}{P}}$

$\cancel{dz + z^2du + \frac{Q^2}{P^2}du = 0}$

$\cancel{z\,dP + 2PQ\,dx + dQ + X\,dx = 0}$

$Q = \alpha P$, $\cancel{z(dP + 2\alpha P\,du) + \alpha\,dP + X\,du}$

$dQ + X\,dx = 0$; $Q = \beta - \alpha x$;

$\frac{dP}{P} + 2Q\,dx = 0$

$lP = -2\beta x + \alpha x^2$

$P = \gamma N^{\alpha x^2 - 2\beta x}$

$\alpha = 0$, $P = \gamma N^{-2\beta x}$

$P\,dx = du = \gamma N^{-2\beta x}dx$

$u = -\frac{\gamma}{2\beta}N^{-2\beta x}$

$N^{-2\beta x} = -\frac{2\beta u}{\gamma} = \frac{P}{\gamma}$;

$dQ + X\,dx = 0$

$dP + 2PQ\,dx = 0$

$dz + z^2P\,dx + \frac{Q^2}{P}dx = 0$ $\qquad dx = \frac{du}{P}$

$P\,dx = du$; $\qquad Q = A - \int X\,dx$

$dz + z^2du + \frac{Q^2}{P^2}du = 0$ $\qquad \frac{dP}{P} = 2dx(A - \int X\,dx)$

$d^2x = 0$ $\qquad P = N^{2\int dx\int X\,dx + \ldots}$

$\frac{d^2u}{du} + \frac{2Q\,du}{P} = 0$ $\qquad \int du = dx\,N^{\int dx\int X\,dx}$

$\frac{Q}{P} = -\frac{d^2u}{2du^2}$

$dz + z^2du + \frac{d^2u^2}{4du^2} = 0$ $\qquad du = P\,dx$

$\frac{dP}{P} = \alpha\,dx$

$\frac{dP}{P} = 2\alpha\,du$

$dz + z^2dx\,N^{-2\int dx\int X\,dx} + dx(X\,dx)N^{+2\int dx\int X\,dx}$ $\qquad lP = \beta - 2\alpha u$

$\frac{Q^2}{P^2} = a^2u^{2n}$; $\quad 2Q\,dx = 2a^2u^{2n}du = \frac{dP}{P}$

$\frac{2Q\,dx}{P} = 2a^2u^{2n}dx = \frac{dP}{P^2}$ $\qquad l\frac{P}{A} = \frac{4a^2u^{\frac{n+2}{2}}}{n+2} = \alpha u^{\frac{n+2}{2}}$

$P = AN^{\alpha u^{\frac{n+2}{2}}}$; $\quad Q = Bu^{\frac{1}{2}n}N^{\alpha u^{\frac{n+2}{2}}}$

$A\,dx = N^{-\alpha u^{\frac{n+2}{2}}}du$; $\quad dQ = BN^{\alpha u^{\frac{n+2}{2}}}du\left(\frac{n}{2}u^{\frac{n-2}{2}} + \frac{n+2}{2}\alpha u^n\right)$

~~n+2=0, A dx = p du~~

Ideoque $\gamma = 0{,}620571$, $1+\gamma = 1{,}620571$, $l.\,\overline{1+\gamma} = 0.2096680$

$$\begin{array}{rr} \text{add. (§. XXXV.)} \log c & \underline{3.5500071} \\ \text{Log}\, v = & 3.7596751 \\ v = & 5750'',0966. \end{array}$$

Ea igitur heic est velocitas adparens caeli directa, ut spatio unius diei percurrat arcum $1^\circ,35',50''$, &c.

§. XXXIX.

Antequam finem huic opellæ impono, tabellam quandam adjicere placet, ut uno quasi obtutu adpareat, quid circa præsentem dissertationis nostræ materiam per observationes ab Auctoribus, qui ad manus fuerunt, indicatas sit compertum, quidve ex calculis nostris prodierit. Designent autem litteræ numeris præfixæ, scil. *a*) De LA LANDE Astr. §. §. 751. 836. cfr. §. 835. *b*) WOLFFII Anf. gr. Astr. §. §. 357. 358. *c*) WINKL. Inst. Math. Phys. §. 1327. *d*) WEIDL. Inst. Math. Astr. §. §. 128. 130. *e*) KEILL Intr. ad ver. Astr. p. 332. 340. 341. cfr. sis WERDRIES Phys. Part. special. Cap. III. § 21. Numeri, quibus nulla littera præfixa est, ostendunt valores calculo repertos.

S. D. G.

	Arcus Retrogr.	Arcus Direct.	Duratio Dir.	Duratio Retrogr.	Duratio Stat.	Temp. interc. duas Retr.
	a. b.) 7° -	- -	b) 244ᵈ -	a. b. d.) 136ᵈ -	b.c.) 8ᵈ -	a.e.) 378ᵈ -
♄	c. d.) 6 -	- -	c) 243 -	c) 140 -	- -	c.) 391 -
	6, 47′	19°,26′	240, 12ʰ	137, 14ʰ	19, 4ʰ,	378, 2ʰ
	a.b.d.) 10 -	- -	b.c.) 284 -	a. b. d.) 119, -	- -	a.) 399 -
	- -	- -	- -	c.) 120 -	b.c.) 4, -	c.) 408 -
♃	- -	- -	- -	- -	- -	e.) 398 -
	9, 57	43, 6	278, 6	120, 15	10, 8	398,21
	a) 12, -	- -	- -	a.b.) 75 -	- -	a.) 789 -
	b.) 10. 12 -	- -	b.c.) 705, -	c.) 73 -	b.c.) 2, -	c.c.) 780 -
♂	d.) 13 -	- -	- -	d.) 57 -	- -	- -
	15, 56	424, 38	707, 5	72, 18	2, 12	779,23
	a) 16, -	- -	- -	a.b.c.d.) 42, -	- -	a.) 584 -
♀	- -	- -	b.c.) 542, -	- -	b.) 1, -	c.) 585 -
	- -	- -	- -	- -	c.) 1, 12	e.) 583 -
	16, 10	591, 7	541, 5	42, 4	- 20	583, 9
	a) 11, -	- -	- -	a.b.c.d.) 22, -	- -	a.) 116 -
☿	- -	- -	b.c.) 93, -	- -	b.c.) - 12	c.) 115 -
	13, 49	128, 2	92, 23	22, 22	- 7	115,21